Financial Modelling and Business Forecasting (FMBF)

Seminar 1: Univariate Time Series Model

DZULFIAN SYAFRIAN

Durham University Business School (DUBS)

dzulfian.syafrian@durham.ac.uk

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Overview

I. Office Hour: Monday 4-5 PM @ Elvet Hill House

II. Seminar Sessions:

- Seminar I: Univariate Time Series: AR, MA, ARMA, Random Walk/RW (and RW with drift), Box-Jenkins, etc.
- Seminar II: Unit root, Cointegration, Engle-Granger, etc.
- Seminar III: Cointegration, ECM, ARCH, etc.
- Seminar IV: GARCH/Forecasting

References

Chris Brooks (2014)
Introductory Econometrics in Finance
Cambridge University Press. 3rd edition.

Chris Brooks (2008)
Introductory Econometrics in Finance. Ch. 5.
Cambridge University Press. 2nd edition.

Richard Harris and Roberts Sollis (2003)
Applied Time Series Modelling and Forecasting
Wiley

Ashish Rajbhandari (2016)
Unit-root tests in Stata
https://blog.stata.com/2016/06/21/unit-root-tests-in-stata/

QUESTION 1: Write down the algebraic form for the following time-series models:

- A stationary AR (1) model with a constant and iid errors with mean zero and variance 1.
- A stationary AR (2) model with a constant, trend and normal iid errors with mean zero and variance 2.
- An MA (2) with a constant and iid errors with mean zero and variance 1. Is it stationary?
- A stationary ARMA (2, 1) with a constant and iid errors with mean zero and variance 1.
- A random walk with iid errors with mean zero and variance 1.

Answers Q1.1 and Q1.2

i. A stationary AR (1) model with a constant and iid errors with mean zero and variance 1.

$$Y_t = \alpha + \beta Y_{t-1} + \epsilon_t \tag{1}$$

where $\epsilon_t \sim$ iid $(0, \sigma_\epsilon^2 = 1)$ and $|\beta| < 1$.

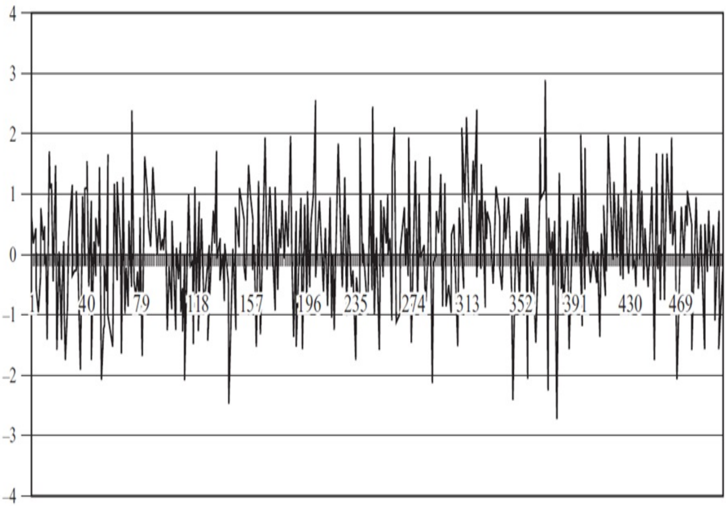
ii. A stationary AR (2) model with a constant, trend and normal iid errors with mean zero and variance 2.

$$Y_t = \alpha + \lambda t + \beta_1 Y_{t-1} + \beta_2 Y_{t-2} + \epsilon_t \tag{2}$$

where $\epsilon_t \sim$ Niid $(0, \sigma_\epsilon^2 = 2)$ and $|\beta| < 1$.

(a) $\beta_1 + \beta_2 < 1$; (b) $\beta_2 - \beta_1 < 1$; and $|\beta_2| < 1$ for trend stationary. We have to add $\lambda = 0$ for stationarity.

White Noise Process



Source: Brooks (2008, p.324)

$|\beta| < 1$ (Figure 5.6) and $|\beta| = 1$ (Figure 5.7)

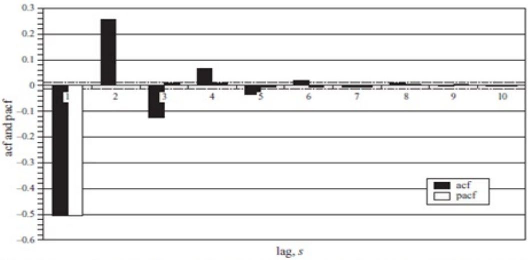


Figure 5.6 Sample autocorrelation and partial autocorrelation functions for a more rapidly decaying AR(1) model with negative coefficient: $y_t = -0.5y_{t-1} + u_t$

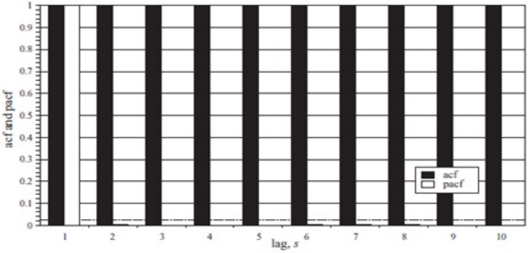


Figure 5.7 Sample autocorrelation and partial autocorrelation functions for a non-stationary model (i.e. a unit coefficient): $y_t = y_{t-1} + u_t$

Answers Q1.3
(Brooks (2008, 212-4); Brooks (2014, p.257-9)

iii. An MA (2) with a constant and iid errors with mean zero and variance 1. Is it stationary?

$$Y_t = \alpha + \theta_1 \epsilon_{t-1} + \theta_2 \epsilon_{t-2} + \epsilon_t \tag{3}$$

where $\epsilon_t \sim$ iid $(0, \sigma_\epsilon^2 = 1)$. It is stationary because:

$E(Y_t) = \alpha$

$Var(Y_t) = \gamma_0 = \sigma_\epsilon^2(1 + \theta_1^2 + \theta_2^2)$

$Cov(Y_t, Y_{t-1}) = \sigma_\epsilon^2(\theta_1 + \theta_1\theta_2)$

$Cov(Y_t, Y_{t-2}) = \sigma_\epsilon^2\theta_2$

and $Cov(Y_t, Y_{t-k}) = 0$ for all $k > 2$

iv. A stationary ARMA (2, 1) with a constant and iid errors with mean zero and variance 1.

$$Y_t = \alpha + \beta_1 Y_{t-1} + \beta_2 Y_{t-2} + \theta_1 \epsilon_{t-1} + \epsilon_t \quad (4)$$

where $\epsilon_t \sim$ iid $(0, \sigma^2_\epsilon = 1)$.

v. A random walk with iid errors with mean zero and variance 1.

$$Y_t = Y_{t-1} + \epsilon_t \quad (5)$$

where $\epsilon_t \sim$ iid $(0, \sigma^2_\epsilon = 1)$.

Questions 2-6

QUESTION 2: Derive the mean, the variance and covariances of the AR (1) process in Question 1.(i).

QUESTION 3: Derive the mean, variance and covariances of the random walk in Question 1.(v). Why is the random walk not stationary? How can you transform a random walk into a stationary process?

QUESTION 4: Derive the mean and variance of a random walk with a constant (drift). Why is the random walk with drift not stationary?

QUESTION 5: Derive the mean and variance of a constant plus a time trend with iid errors with mean zero and variance 1.

QUESTION 6: Explain Box-Jenkins methodology for univariate time series modelling.

Answers Q2 (Brooks (2008, 218-22); Brooks (2014, p.263-6)

QUESTION 2: Derive the mean, the variance and covariances of the AR (1) process in Question 1.(i).

Recall equation at Q1.1: $Y_t = \alpha + \beta Y_{t-1} + \epsilon_t$

The expected mean value:

$$E(Y_t) = \alpha(1 + \beta + \beta^2 + ...) = \frac{\alpha}{1-\beta} = \mu \tag{6}$$

The variance:

$$Var(Y_t) = \sigma_\epsilon^2(1 + \beta^2 + \beta^4 + ...) = \frac{\sigma_\epsilon^2}{1-\beta^2} = \sigma_y^2 = \gamma_0 \tag{7}$$

Covariance:

$\mathrm{Cov}(\mathrm{Y}_t, Y_{t-1}) = \gamma_1 = \beta\sigma_y^2$; $Cov(Y_t, Y_{t-2}) = \gamma_2 = \beta^2\sigma_y^2$;

$$Cov(Y_t, Y_{t-k}) = \gamma_k = \beta^k\sigma_y^2 \tag{8}$$

Answers Q3

QUESTION 3: Derive the mean, variance of the RW in Q1.(iii). is the RW not stationary? How to transform a RW into a stationary process?

Recall (5): $Y_t = Y_{t-1} + \epsilon_t$, where $\epsilon_t \sim$ iid $(0, \sigma_\epsilon^2 = 1)$, by iterative substitution we obtain:

$$Y_t = Y_0 + \epsilon_1 + \epsilon_2 + ... + \epsilon_t, E(Y_t) = Y_0 \tag{9}$$

Equation (9) shows that the mean is constant over time.

The variance:

$$Var(Y_t) = Var(Y_0) + Var(\epsilon_1) + Var(\epsilon_2) + ... + Var(\epsilon_t) = t\sigma_\epsilon^2 \tag{10}$$

Equation (10) shows that the variance is non-stationary because its variance varies with time. We can transform a random walk into a stationary process by differencing it,

$$(Y_t - Y_{t-1} = \epsilon_t) = (\Delta Y_t = \epsilon_t) \tag{11}$$

QUESTION 4: Derive the mean and variance of the RW with a constant (drift). is the RW with drift not stationary?

$Y_t = \alpha + Y_{t-1} + \epsilon_t$, where $\epsilon_t \sim$ iid $(0, \sigma_\epsilon^2 = 1)$, by iterative substitution we obtain:

$$Y_t = Y_0 + t\alpha + \epsilon_1 + \epsilon_2 + ... + \epsilon_t, E(Y_t) = Y_0 + \alpha t \tag{12}$$

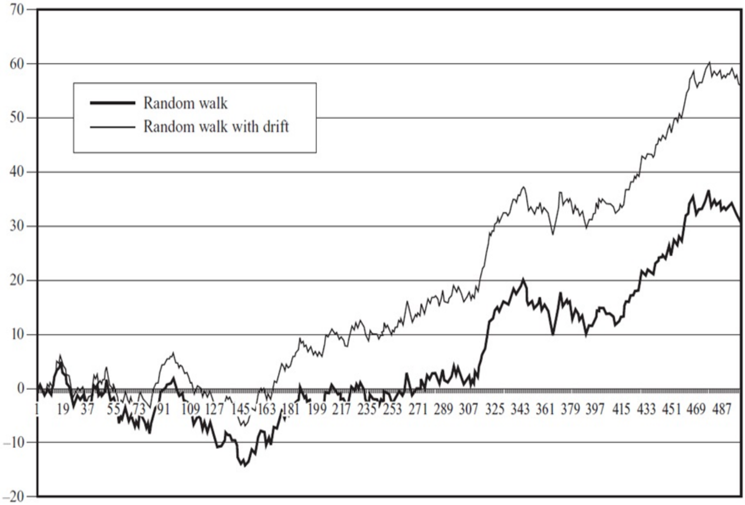
Equation (12) shows that the mean is not constant over time.

The variance:

$$Var(Y_t) = Var(Y_0) + Var(t\alpha) + Var(\epsilon_1) + Var(\epsilon_2) + ... + Var(\epsilon_t) = t\sigma_\epsilon^2 \tag{13}$$

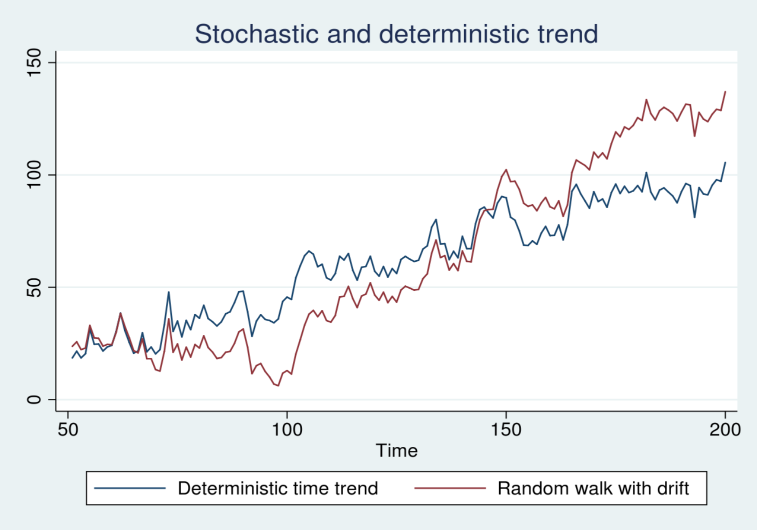
Equation (12) and (13) shows that the this RW is non-stationary because the mean and variance vary with time.

Random Walk/RW (Without Drift) and RW with Drift



Source: Brooks (2008, p.324)

Random Walk With Drift vs Time trend



Source: Rajbhandari,2016

QUESTION 5: Derive the mean and variance of a constant plus a time trend with iid errors with mean zero and variance 1. Is the process stationary?

$Y_t = \alpha + t\beta + \epsilon_t$, where $\epsilon_t \sim$ iid $(0, \sigma_\epsilon^2 = 1)$, by the same process we obtain the general form:

$$Y_t = Y_0 + \alpha t + \epsilon_t, E(Y_t) = Y_0 + \alpha t \tag{14}$$

Equation (14) shows that the mean is not constant over time.

The variance:

$$Var(Y_t) = \sigma_\epsilon^2 \tag{15}$$

Equation (14) shows that this process is non-stationary because the mean varies with time.

Answers Q6: Lecture 1: slides 38-9

QUESTION 6: Explain Box-Jenkins methodology for univariate time series modelling.

Based on Box and Jenkins (1976): propose a method to estimate ARMA model systematically by doing 3 steps:

1. **Identification**: Determine the order by plotting data overtime and ACF and PACF.

2. **Estimation**: estimate the parameter. Can be used either least square or maximum likelihood methods.

3. **Diagnostics checking**: Can use 2 methods, overfitting and residual diagnostics (more common). Are the residuals free of autocorrelation?

- Three main selection criteria: AIC, SBIC and HQIC.

More detail read: Brooks (2008, p.230-3) or Brooks (2014, p.273-6)

Conclusions-1

Q	Model	Form
1.1	AR(1)	$Y_t = \alpha + \beta Y_{t-1} + \epsilon_t$
1.2	AR(2) (+) trend	$Y_t = \alpha + \lambda t + \beta_1 Y_{t-1} + \beta_2 Y_{t-2} + \epsilon_t$
1.3	MA(2)	$Y_t = \alpha + \theta_1 \epsilon_{t-1} + \theta_2 \epsilon_{t-2} + \epsilon_t$
1.4	ARMA(2,1)	$Y_t = \alpha + \beta_1 Y_{t-1} + \beta_2 Y_{t-2} + \theta_1 \epsilon_{t-1} + \epsilon_t$
1.5	R.W.	$Y_t = \alpha + Y_{t-1} + \epsilon_t$
3	R.W.(-) α	$Y_t = Y_{t-1} + \epsilon_t$
4	R.W.(+) α	$Y_t = \alpha + Y_{t-1} + \epsilon_t$
5	Trend (+) α	$Y_t = \alpha + \beta t + \epsilon_t$

Conclusions-2

Q	Model	Mean	Variance	Stationary
1.1	AR(1)	$\frac{\alpha}{1-\beta}$	$\frac{\sigma_\epsilon^2}{1-\beta^2} = \sigma_y^2 = \gamma_0$	Yes, if $\lvert\beta\rvert < 1$
1.2	AR(2)	-	-	-
1.3	MA(2) (+) trend	α	$\sigma_\epsilon^2(1 + \theta_1^2 + \theta_2^2) = \gamma_0$	Yes
1.4	ARMA(2,1)	-	-	-
1.5	R.W.	-	tσ_ϵ^2	No
3	R.W.(-) α	Y$_0$	tσ_ϵ^2	No
4	R.W.(+) α	Y$_0 + \alpha t$	tσ_ϵ^2	No
5	Trend (+) α	Y$_0 + \alpha t$	σ_ϵ^2	No