$$1 + b + b^2 + b^3 + \cdots + b^n + \cdots = \frac{1}{1-b} \quad \text{if } |b| < 1$$

$$(1-b)^{-1} = \frac{1}{1-b} = 1 + b + b^2 + b^3 + \cdots + b^n + \cdots \quad \text{if } |b| < 1$$

$$AR(1): y_t = \mu + \phi_1 y_{t-1} + u_t$$

$$= \mu + \phi_1 L y_t + u_t$$

$$1 \cdot y_t - \phi_1 L y_t = \mu + u_t$$

$$(1 - \phi_1 L) y_t = \mu + u_t$$

$$\underbrace{(1-\phi_1 L)^{-1}(1-\phi_1 L)}_{=1} y_t = (1-\phi_1 L)^{-1}(\mu + u_t)$$

$$(1-\phi_1 L)^{-1} = ?$$

$$(1-b)^{-1} = 1 + b + b^2 + \cdots + b^n + \cdots$$

$$(1-\phi_1 L)^{-1} = 1 + \phi_1 L + \phi_1^2 L^2 + \phi_1^3 L^3 + \cdots + \phi_1^n L^n + \cdots$$

if $|\phi_1| < 1$

$$y_t = (1 + \phi_1 L + \phi_1^2 L^2 + \phi_1^3 L^3 + \cdots + \phi_1^n L^n + \cdots)(\mu + u_t)$$

$$= (1 + \phi_1 L + \phi_1^2 L^2 + \cdots)\mu + (1 + \phi_1 L + \phi_1^2 L^2 + \cdots) u_t$$

$$= \mu + \phi_1 L \mu + \phi_1^2 L^2 \mu + \cdots + u_t + \phi_1 L u_t + \phi_1^2 L^2 u_t + \cdots$$

$$= \mu + \phi_1 \mu + \phi_1^2 \cdot \mu + \cdots + u_t + \phi_1 u_{t-1} + \phi_1^2 u_{t-2} + \cdots$$

$$= \underbrace{(1 + \phi_1 + \phi_1^2 + \cdots)}_{\frac{1}{1-\phi_1}} \mu + u_t + \phi_1 u_{t-1} + \phi_1^2 u_{t-2} + \cdots$$

$|\phi_1| < 1$

$$= \frac{1}{1-\phi_1} \cdot \mu + \underbrace{u_t + \phi_1 u_{t-1} + \phi_1^2 u_{t-2} + \cdots}_{MA(\infty)}$$