FMBF: Computer lab 3

This session covers ARCH and GARCH models. We will see how to estimate the various models and perform appropriate diagnostic tests to assist in choosing a preferred model.

We will work with a dataset comprising exchange rate data for the period 2 January 1980 to 21 May 1987. The contents of this dataset are listed in following table.

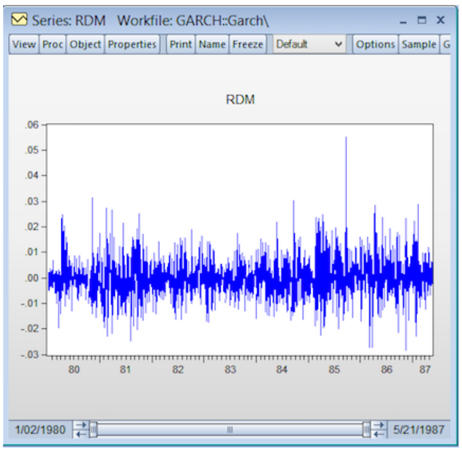
Variable	Description
BP	US$ - British Pound
CD	US$ - Canadian Dollar
DM	US$ - Deutsche Mark
JY	US$ - Japanese Yen
SF	US$ - Swiss Franc

ARCH/GARCH Modeling

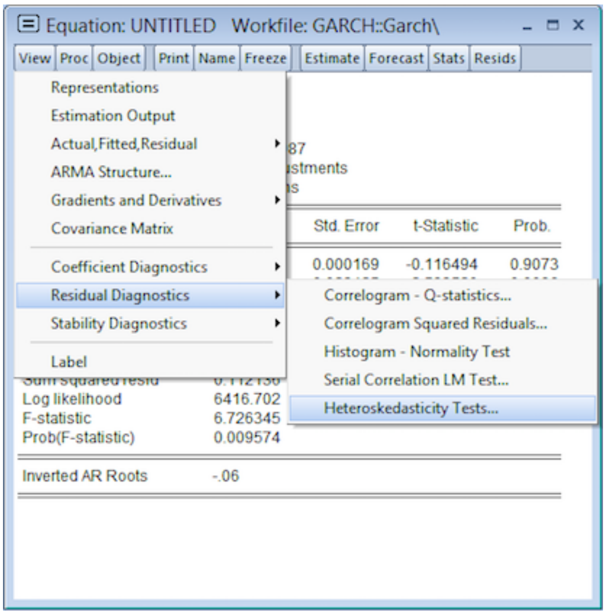
We will initially test for ARCH effects. Subsequently, we will construct ARCH/GARCH models. Finally, we will make forecasting based on the model we constructed.

Testing for 'ARCH effects'

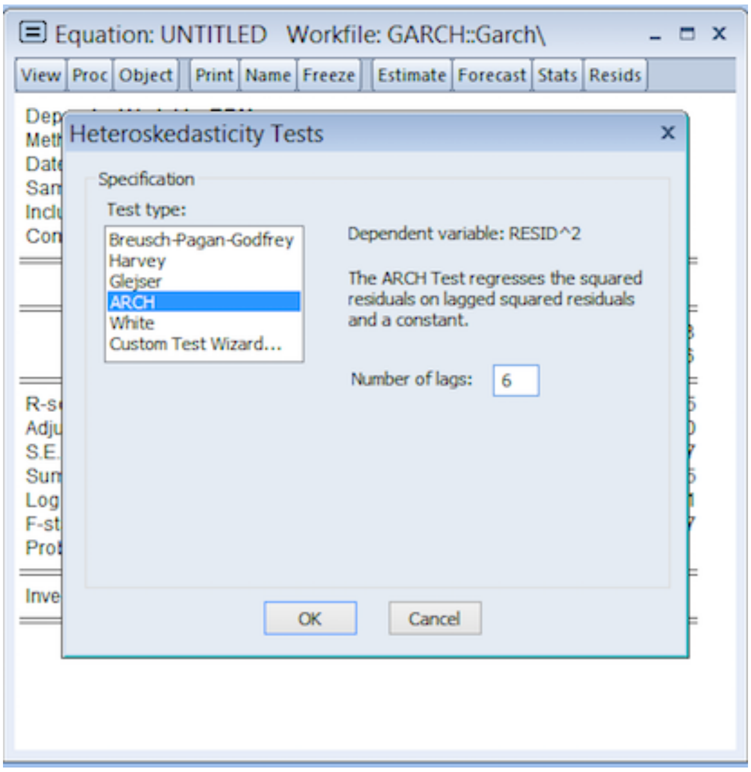
1. Download the data 'garch.xls' from duo and load into EViews.
2. To calculate exchange rate returns, dlog all of the exchange rate series. Name the newly created variables as rbp, rcd, rdm, rjy, and rsf.
3. Plot the daily change data (exchange rate returns) in EViews; what conclusions do you come to? For example, the line graph of 'rdm' calculated as dlog(DM) is shown as follows.



4. Identify an appropriate ARMA model for each of the exchange rate return series by following the Box–Jenkins methodology (Please check Computer lab 1).
5. We use 'rdm' as an example to test ARCH effects. Estimate a AR(1) model for rdm. Then, in the Equation window, select View > Residual Diagnostics > Heteroskedasticity Test



Select ARCH, and Input '6' in the 'Number of lags' box. We just show an example of test with 6 lagged terms. Please check the note of first computer lab on how to choose optimum number of lags.



Note:

A test for the presence of ARCH in the residuals is calculated by regressing the squared residuals on a constant and p lags, where p is set by the user. Therefore, to test the ARCH effects, we should firstly estimate a linear model to get residuals.

The AR(1) model has been chosen entirely arbitrarily at this stage to show a example. However, you should carefully choose the type and order of the model in practice, since the variance is measured around the mean and therefore any mis-specification in the mean is likely to lead to a mis-specified variance.

6. Finally, we get the results. The null hypothesis of the test is that there is no ARCH effect. Both F-test and Lagrange multiplier test (Chi-Square) show significant results, indicating that ARCH effects exist in ‘rdm’ series.

Equation: UNTITLED Workfile: GARCH::Garch\

View | Proc | Object | Print | Name | Freeze | Estimate | Forecast | Stats | Resids

Heteroskedasticity Test: ARCH

F-statistic	13.76245	Prob. F(6,1852)	0.0000
Obs*R-squared	79.34891	Prob. Chi-Square(6)	0.0000

Test Equation:
Dependent Variable: RESID^2
Method: Least Squares
Date: 01/08/15 Time: 00:30
Sample (adjusted): 1/14/1980 5/21/1987
Included observations: 1859 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	3.63E-05	4.01E-06	9.043649	0.0000
RESID^2(-1)	0.072848	0.023044	3.161237	0.0016
RESID^2(-2)	0.051576	0.023058	2.236758	0.0254
RESID^2(-3)	0.038979	0.023068	1.689751	0.0912
RESID^2(-4)	0.042468	0.023068	1.840974	0.0658
RESID^2(-5)	0.064232	0.023058	2.785648	0.0054
RESID^2(-6)	0.128593	0.023043	5.580464	0.0000

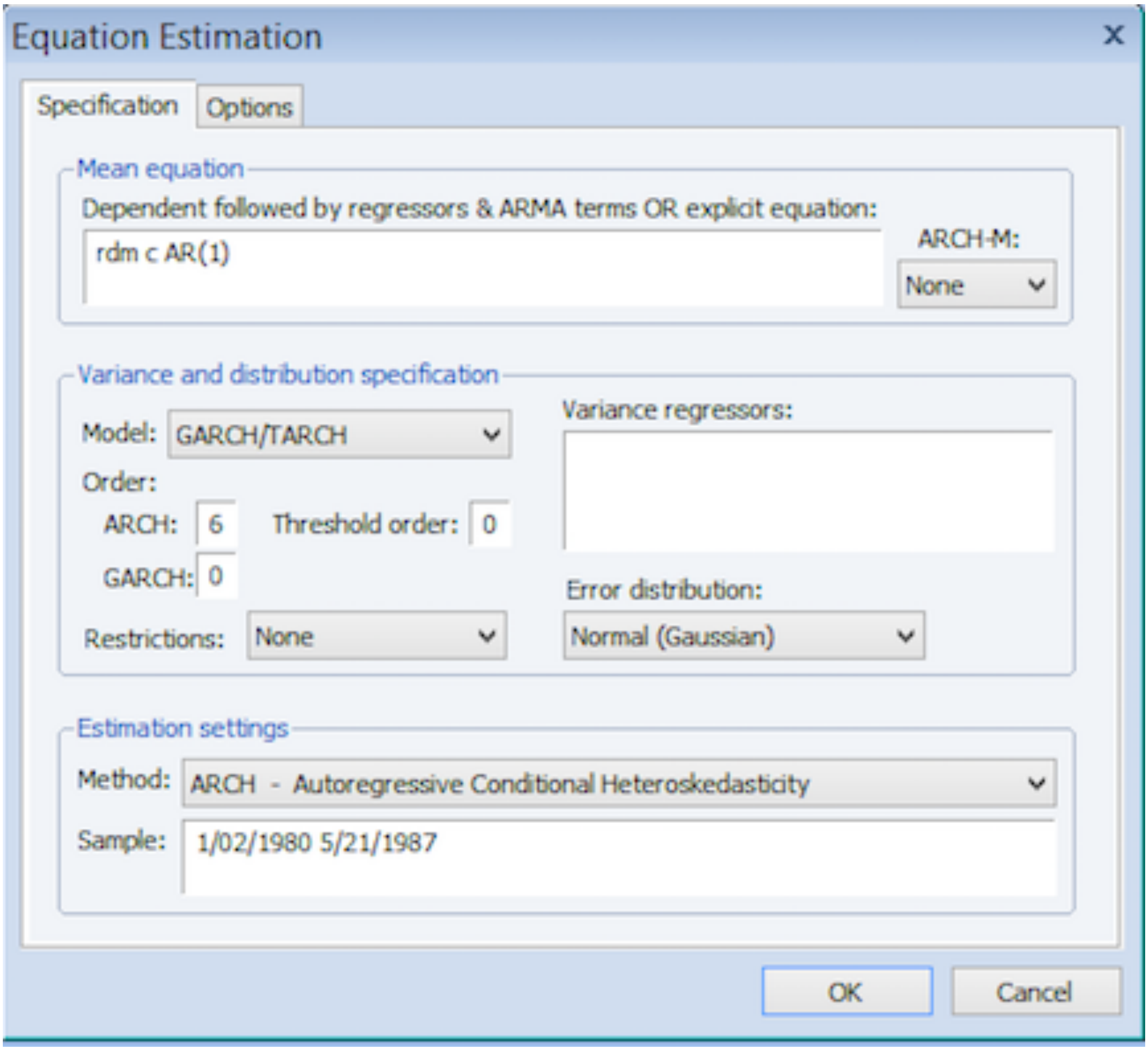
R-squared	0.042684	Mean dependent var	6.03E-05
Adjusted R-squared	0.039582	S.D. dependent var	0.000126
S.E. of regression	0.000123	Akaike info criterion	-15.16421
Sum squared resid	2.80E-05	Schwarz criterion	-15.14339

Estimating ARCH/GARCH models

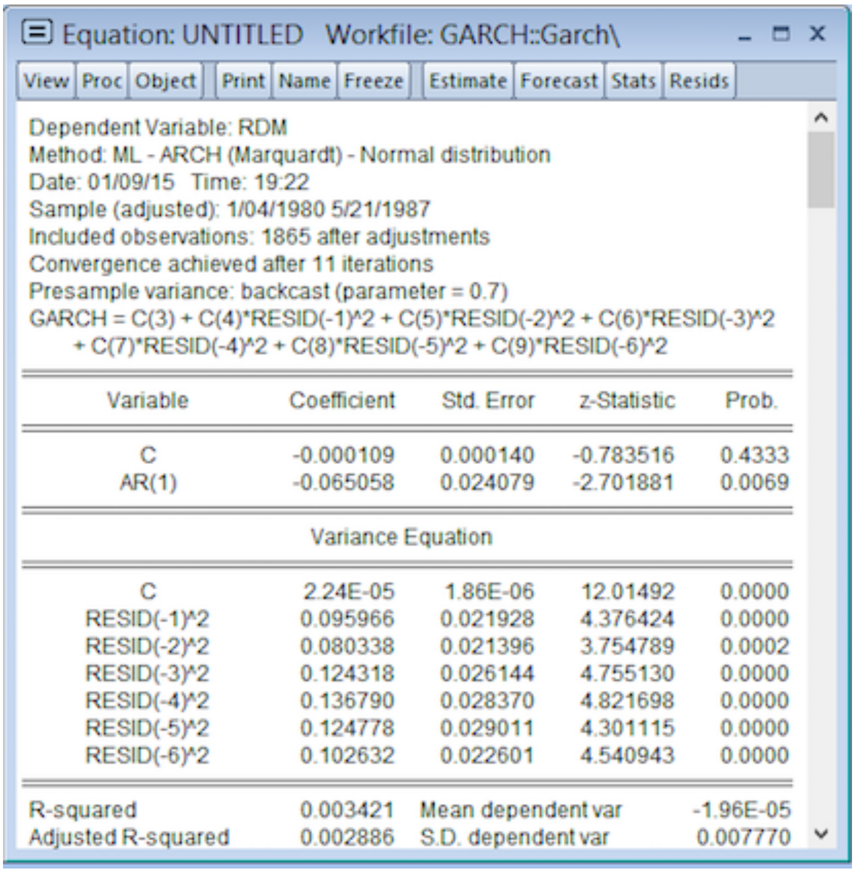
In this section, we will use EViews to estimate ARCH model, the basic GARCH model, asymmetric GARCH models (GJR and EGARCH), and GARCH-in-mean model.

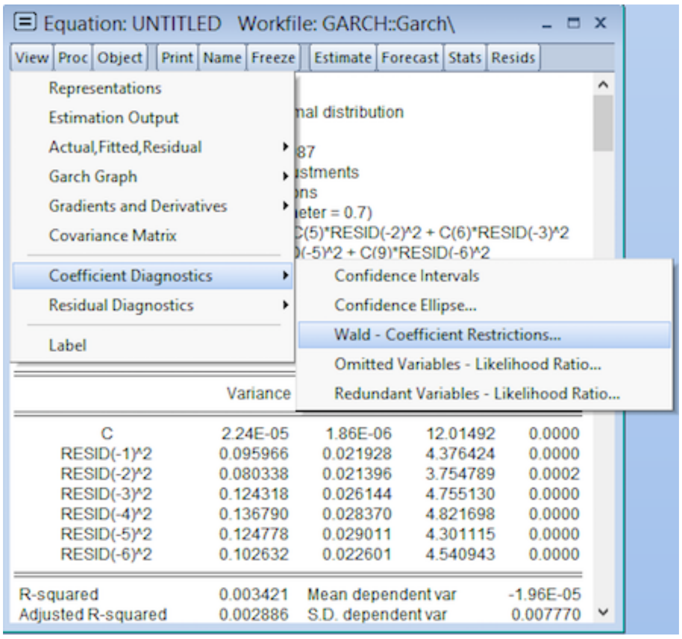
ARCH

1. Estimate an appropriate ARCH model based on your results from the previous step. To do this in EViews, go to 'Quick > Estimate Equation'. Then, select 'ARCH – Autoregressive Conditional Heteroskedasticity'.
2. In 'Equation Estimation' window, input your identified ARMA model in the 'Mean equation' box. Here, we use AR(1) model as an example. In practice, please carefully identify an appropriate ARMA model. In the 'Variance and distribution specification' box, input the appropriate specification. For example, if you want to estimate a ARCH(6) model, you should input '6' for ARCH order, and '0' for GARCH order.



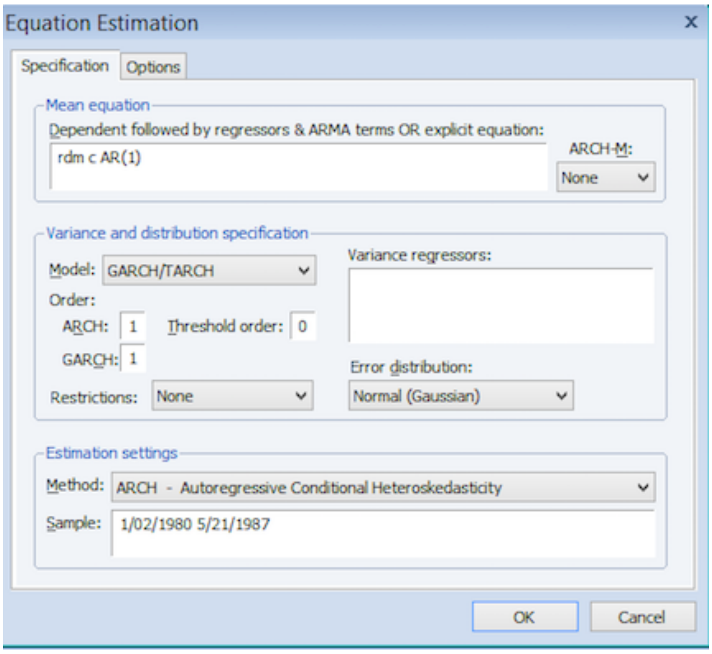
3. Analyze the results from your model. In particular, you will want to check if any of the parameters are insignificant, and if so consider a more parsimonious specification. It may be that you want to perform a Wald test if you have more than one insignificant parameter. This can be done by View > Coefficient Diagnostics > Wald – Coefficient Restrictions. For more information on Wald test, please check computer lab 1.



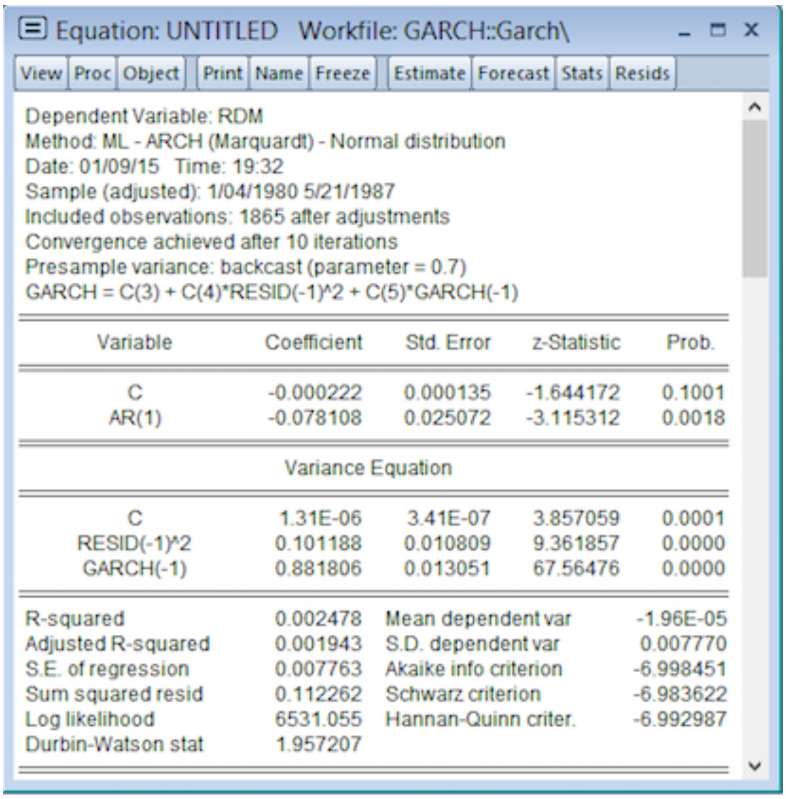


GARCH

1. We will now see if it is advantageous to formulate a more parsimonious GARCH(1,1) model. Using the same AR specification identified above (we still use AR(1) model as an example) as the mean equation, go to the 'Equation Estimation' window, but this time specify both ARCH and GARCH order as '1', so we have a GARCH(1,1) model.



2. Interpret the results. Consider if you can improve the model with a change in the ARMA structure.



Equation: UNTITLED Workfile: GARCH::Garch\

View | Proc | Object | Print | Name | Freeze | Estimate | Forecast | Stats | Resids

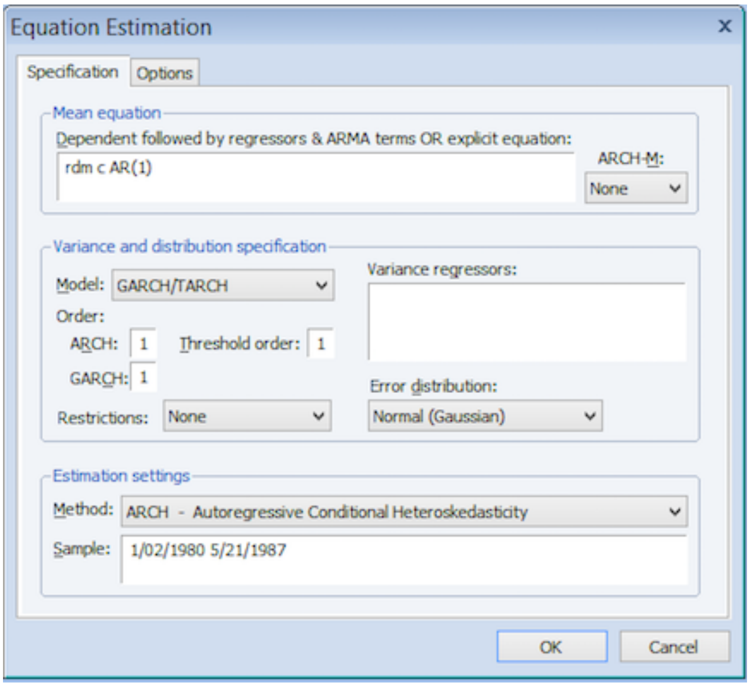
Dependent Variable: RDM
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 01/09/15 Time: 19:32
Sample (adjusted): 1/04/1980 5/21/1987
Included observations: 1865 after adjustments
Convergence achieved after 10 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	-0.000222	0.000135	-1.644172	0.1001
AR(1)	-0.078108	0.025072	-3.115312	0.0018
		Variance Equation		
C	1.31E-06	3.41E-07	3.857059	0.0001
RESID(-1)^2	0.101188	0.010809	9.361857	0.0000
GARCH(-1)	0.881806	0.013051	67.56476	0.0000

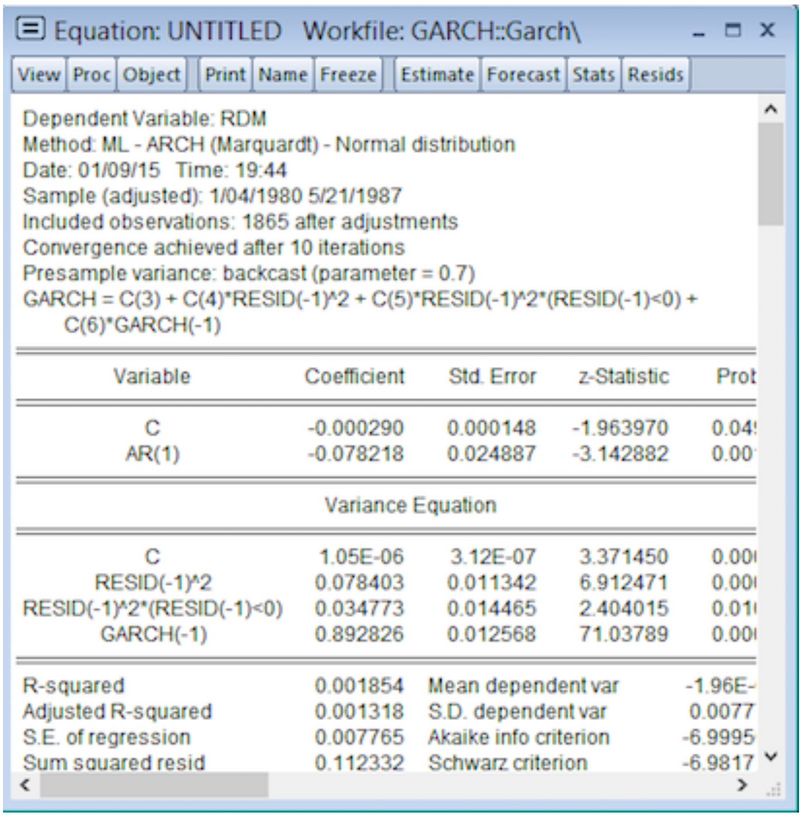
R-squared	0.002478	Mean dependent var	-1.96E-05
Adjusted R-squared	0.001943	S.D. dependent var	0.007770
S.E. of regression	0.007763	Akaike info criterion	-6.998451
Sum squared resid	0.112262	Schwarz criterion	-6.983622
Log likelihood	6531.055	Hannan-Quinn criter.	-6.992987
Durbin-Watson stat	1.957207		

GJR-GARCH

1. Estimate a GJR-GARCH(1,1) model using the ARMA specification you identified above as the mean equation (we still use AR(1) model as an example). To do this, open ‘Equation Estimation’ window, and change the ‘Threshold order’ number from 0 to 1.



2. Interpret the results.



Dependent Variable: RDM
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 01/09/15 Time: 19:44
Sample (adjusted): 1/04/1980 5/21/1987
Included observations: 1865 after adjustments
Convergence achieved after 10 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*RESID(-1)^2*(RESID(-1)<0) + C(6)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob
C	-0.000290	0.000148	-1.963970	0.04
AR(1)	-0.078218	0.024887	-3.142882	0.00
Variance Equation				
C	1.05E-06	3.12E-07	3.371450	0.00
RESID(-1)^2	0.078403	0.011342	6.912471	0.00
RESID(-1)^2*(RESID(-1)<0)	0.034773	0.014465	2.404015	0.01
GARCH(-1)	0.892826	0.012568	71.03789	0.00

R-squared	0.001854	Mean dependent var	-1.96E-
Adjusted R-squared	0.001318	S.D. dependent var	0.0077
S.E. of regression	0.007765	Akaike info criterion	-6.9995
Sum squared resid	0.112332	Schwarz criterion	-6.9817

Note:

Due to Glosten, Jaganathan and Runkle

$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta\sigma_{t-1}^2 + \gamma u_{t-1}^2 I_{t-1}$

where $I_{t-1} = 1$ if $u_{t-1} < 0$

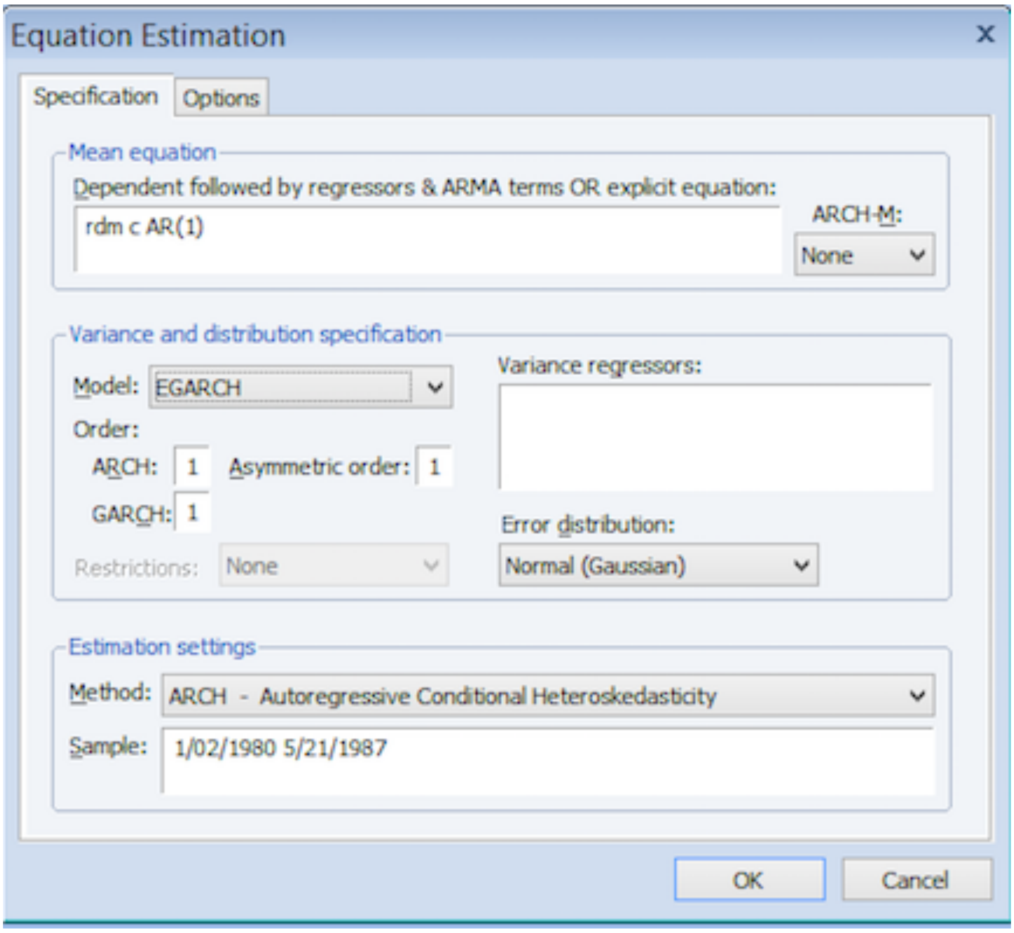
$= 0$ otherwise

For a leverage effect, we would see $\gamma > 0$

We require $\alpha_1 + \gamma \geq 0$ and $\alpha_1 \geq 0$ for non-negativity.

EGARCH

1. Estimate an EGARCH(1,1) model using the ARMA specification you identified above as the mean equation (we still use AR(1) model as an example). To do this, open 'Equation Estimation' window, and select 'EGARCH' in the 'Variance and distribution specification' box.



2. Interpret results.

Equation: UNTITLED   Workfile: GARCH::Garch\

View | Proc | Object | Print | Name | Freeze | Estimate | Forecast | Stats | Resids

Dependent Variable: RDM
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 01/09/15   Time: 19:51
Sample (adjusted): 1/04/1980 5/21/1987
Included observations: 1865 after adjustments
Convergence achieved after 11 iterations
Presample variance: backcast (parameter = 0.7)
LOG(GARCH) = C(3) + C(4)*ABS(RESID(-1)/@SQRT(GARCH(-1))) + C(5)
*RESID(-1)/@SQRT(GARCH(-1)) + C(6)*LOG(GARCH(-1))

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	-0.000314	0.000145	-2.169099	0.0301
AR(1)	-0.082236	0.024145	-3.405867	0.0007
		Variance Equation		
C(3)	-0.418916	0.075316	-5.562085	0.0000
C(4)	0.196645	0.019921	9.871341	0.0000
C(5)	-0.019000	0.009381	-2.025374	0.0428
C(6)	0.972646	0.006865	141.6762	0.0000

R-squared	0.001425	Mean dependent var	-1.96E-05
Adjusted R-squared	0.000889	S.D. dependent var	0.007770
S.E. of regression	0.007767	Akaike info criterion	-7.002333
Sum squared resid	0.112381	Schwarz criterion	-6.984539
Log likelihood	6535.676	Hannan-Quinn criter.	-6.995776

Note:

Suggested by Nelson (1991). The variance equation is given by

$$\log(\sigma_t^{\,2}) = \omega + \beta \log(\sigma_{t-1}^{\,2}) + \gamma \frac{u_{t-1}}{\sqrt{\sigma_{t-1}^{\,2}}} + \alpha \left[\frac{|u_{t-1}|}{\sqrt{\sigma_{t-1}^{\,2}}} - \sqrt{\frac{2}{\pi}} \right]$$

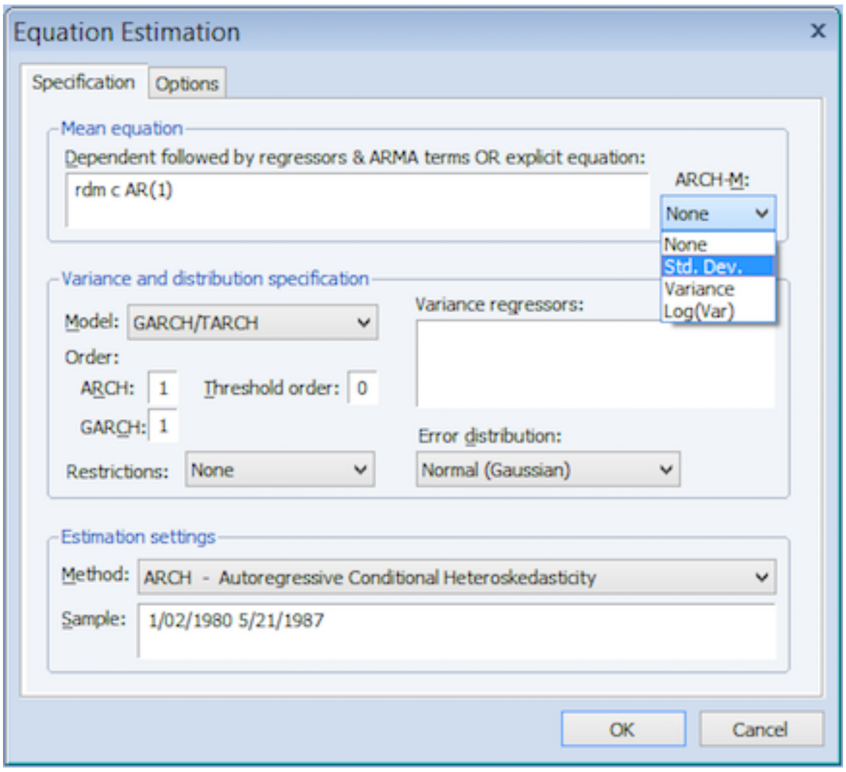
Advantages of the model:

Since we model the $\log(\sigma_t^2)$, then even if the parameters are negative, σ_t^2 will be positive.

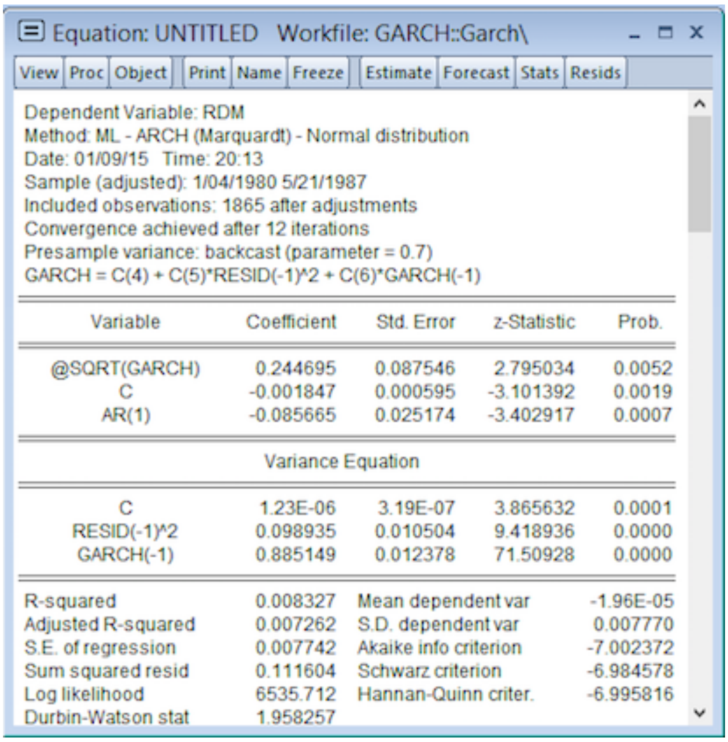
We can account for the leverage effect: if the relationship between volatility and returns is negative, γ will be negative.

GARCH-in-mean

1. Estimate a GARCH(1,1)-in-Mean using the ARMA specification you identified above as the mean equation (we still use AR(1) model as an example). To do this, open 'Equation Estimation window', and select appropriate option in the 'ARCH-M' box to add the conditional standard deviation, the conditional variance, or the log of the conditional variance to the mean equation.



2. Interpret the results.



Equation: UNTITLED Workfile: GARCH::Garch\

View | Proc | Object | Print | Name | Freeze | Estimate | Forecast | Stats | Resids

Dependent Variable: RDM
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 01/09/15 Time: 20:13
Sample (adjusted): 1/04/1980 5/21/1987
Included observations: 1865 after adjustments
Convergence achieved after 12 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(4) + C(5)*RESID(-1)^2 + C(6)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
@SQRT(GARCH)	0.244695	0.087546	2.795034	0.0052
C	-0.001847	0.000595	-3.101392	0.0019
AR(1)	-0.085665	0.025174	-3.402917	0.0007
		Variance Equation		
C	1.23E-06	3.19E-07	3.865632	0.0001
RESID(-1)^2	0.098935	0.010504	9.418936	0.0000
GARCH(-1)	0.885149	0.012378	71.50928	0.0000

R-squared	0.008327	Mean dependent var	-1.96E-05
Adjusted R-squared	0.007262	S.D. dependent var	0.007770
S.E. of regression	0.007742	Akaike info criterion	-7.002372
Sum squared resid	0.111604	Schwarz criterion	-6.984578
Log likelihood	6535.712	Hannan-Quinn criter.	-6.995816
Durbin-Watson stat	1.958257		

In above results, the conditional standard deviation (@SQRT(GARCH)) is added in the mean equation.

Equation: UNTITLED Workfile: GARCH::Garch\

View | Proc | Object | Print | Name | Freeze | Estimate | Forecast | Stats | Resids

Dependent Variable: RDM
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 01/09/15 Time: 20:15
Sample (adjusted): 1/04/1980 5/21/1987
Included observations: 1865 after adjustments
Convergence achieved after 13 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(4) + C(5)*RESID(-1)^2 + C(6)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
GARCH	15.71648	5.904249	2.661892	0.0078
C	-0.000969	0.000304	-3.186633	0.0014
AR(1)	-0.086602	0.025170	-3.440729	0.0006
		Variance Equation		
C	1.24E-06	3.21E-07	3.870716	0.0001
RESID(-1)^2	0.098954	0.010479	9.442854	0.0000
GARCH(-1)	0.884995	0.012385	71.45897	0.0000

R-squared	0.007702	Mean dependent var	-1.96E-05
Adjusted R-squared	0.006636	S.D. dependent var	0.007770
S.E. of regression	0.007744	Akaike info criterion	-7.002194
Sum squared resid	0.111674	Schwarz criterion	-6.984400
Log likelihood	6535.546	Hannan-Quinn criter.	-6.995638
Durbin-Watson stat	1.958761		

In above results, the conditional variance (GARCH) is added in the mean equation.

Equation: UNTITLED Workfile: GARCH::Garch\

View | Proc | Object | Print | Name | Freeze | Estimate | Forecast | Stats | Resids

Dependent Variable: RDM
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 01/09/15 Time: 20:15
Sample (adjusted): 1/04/1980 5/21/1987
Included observations: 1865 after adjustments
Convergence achieved after 10 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(4) + C(5)*RESID(-1)^2 + C(6)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
LOG(GARCH)	0.000821	0.000299	2.744274	0.0061
C	0.008070	0.003019	2.673102	0.0075
AR(1)	-0.085285	0.025114	-3.395921	0.0007
		Variance Equation		
C	1.24E-06	3.21E-07	3.870447	0.0001
RESID(-1)^2	0.098684	0.010520	9.380760	0.0000
GARCH(-1)	0.885182	0.012436	71.17952	0.0000

R-squared	0.008179	Mean dependent var	-1.96E-05
Adjusted R-squared	0.007114	S.D. dependent var	0.007770
S.E. of regression	0.007743	Akaike info criterion	-7.002208
Sum squared resid	0.111620	Schwarz criterion	-6.984414
Log likelihood	6535.559	Hannan-Quinn criter.	-6.995652
Durbin-Watson stat	1.956490		

In above results, the log of the conditional variance (LOG(GARCH)) is added in the mean equation.

Note:

Engle, Lilien and Robins (1987) suggested the ARCH-M specification. A GARCH-M model would be:

$$y_t = \mu + \delta\sigma_{t-1} + u_t \quad , u_t \sim \mathrm{N}(0,\sigma_t^2)$$
$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta\sigma_{t-1}^{\;2}$$

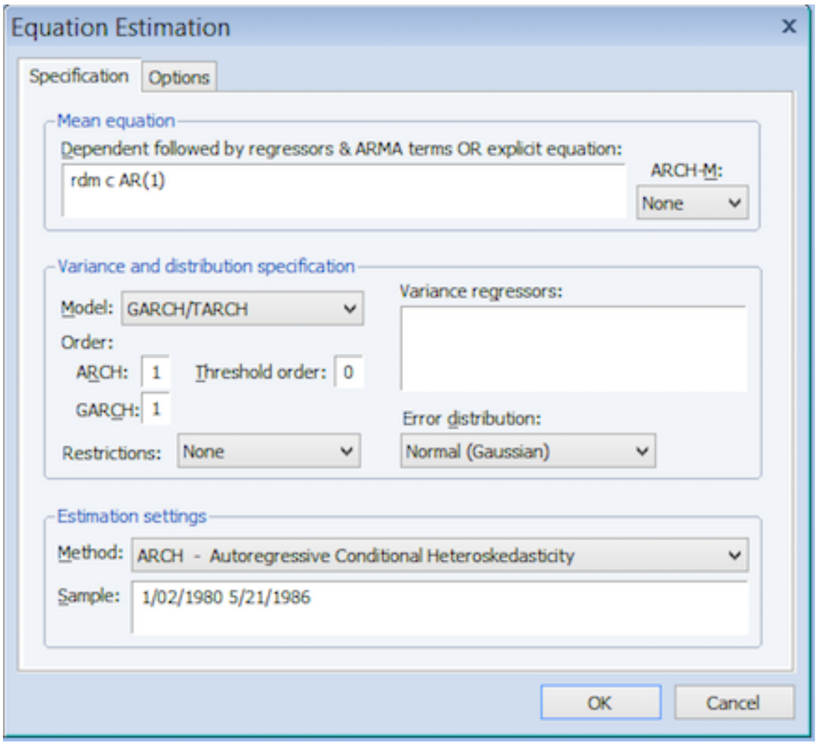
If δ is positive and statistically significant, then increased risk, given by an increase in the conditional variance, leads to a rise in the mean return. Thus δ can be interpreted as a risk premium.

In our case, the estimated parameter on the mean equation has a positive sign and is statistically significant. We would thus conclude that for the series 'rdm', there is a positive feedback from the conditional variance to the conditional mean. In other words, higher risk lead to higher returns.

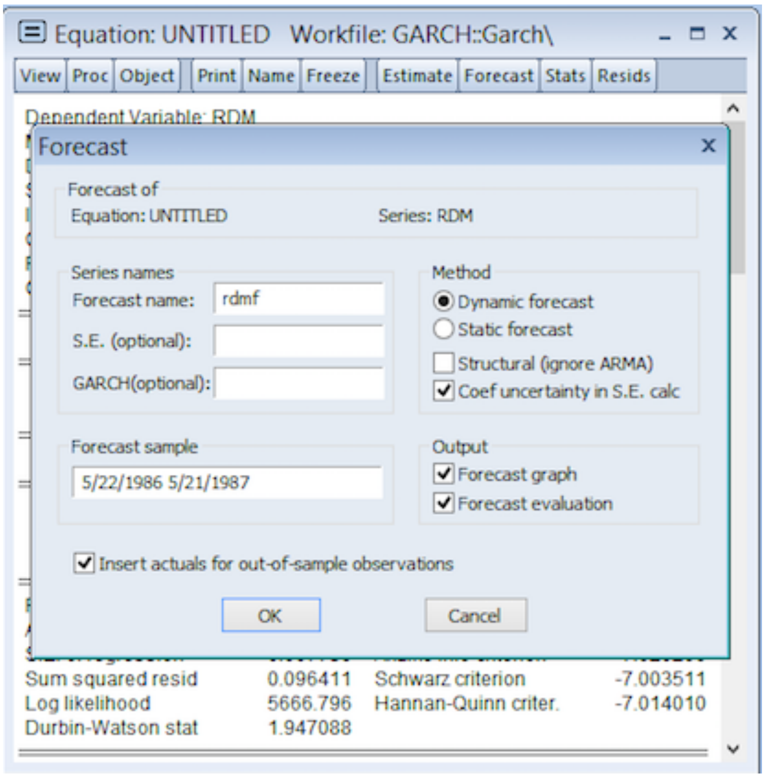
Forecasting from GARCH models

We will use preferred ARMA(p,q)-GARCH(1,1) model (we use AR(1)-GARCH(1,1) as an example) to make forecasting.

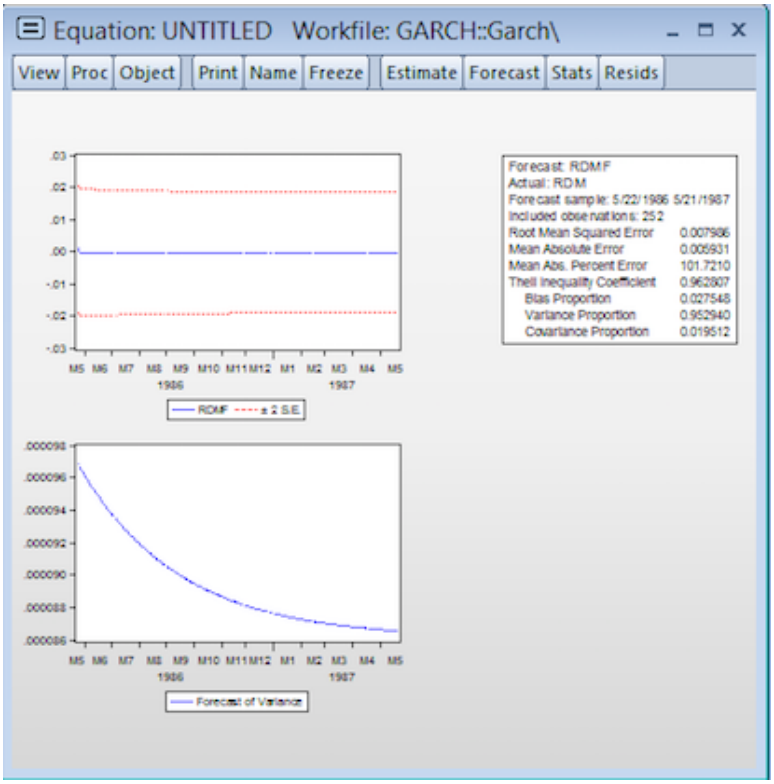
1. We use only a sub-sample of available data for model estimation. Suppose, for example, we stopped the estimation of the GARCH(1,1) model for the series 'rdm' at 21st May 1986 so as to keep the last one year of data for forecasting.



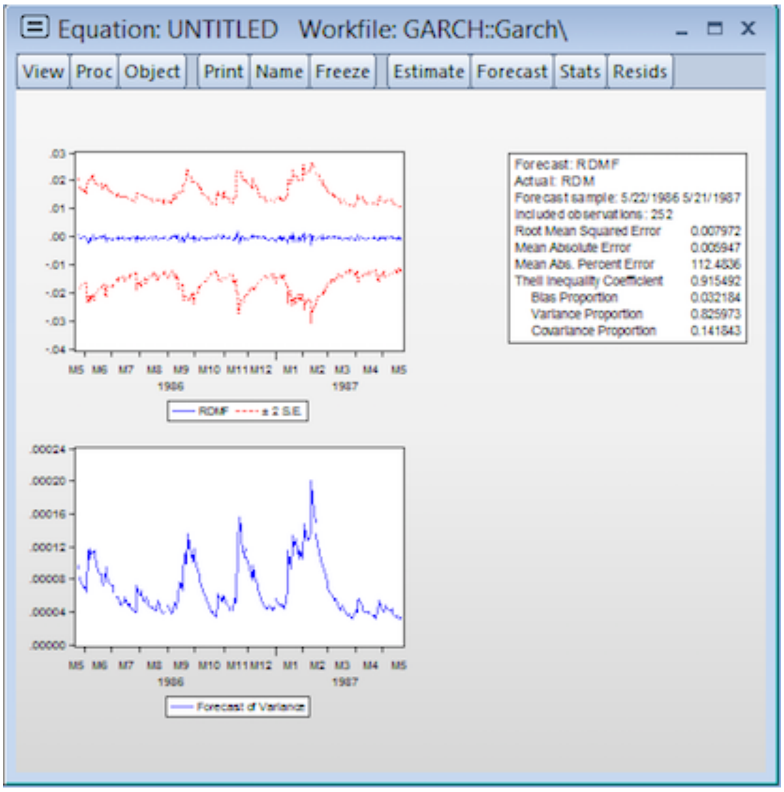
2. In the ‘Equation’ window, click on the ‘Forecast’. In the ‘Forecast’ window, adjust the ‘Forecast sample’ as ‘5/22/1986 5/21/1987’. We can choose to produce static (a series of rolling single-step-ahead) or dynamic (multiple-step-ahead) forecasts.



3. Finally, we get following results. Examine the forecasts from your model.



Dynamic forecasts of the conditional variance



Static forecasts of the conditional variance

References

Brooks, Chris (2008) *Introductory Econometrics for Finance, 2nd edition*. Cambridge University Press.

Engle, R. F., Lilien, D. M. and Robins, R. P. (1987) Estimating Time Varying Risk Premia in the Term Structure: the ARCH-M Model, *Econometrica* 55(2), 391–407

Glosten, L. R., Jagannathan, R. and Runkle, D. E. (1993) On the Relation Between the Expected Value and the Volatility of the Nominal Excess Return on Stocks, *The Journal of Finance* 48(5), 1779–801

Nelson, D. B. (1991) Conditional Heteroskedasticity in Asset Returns: a New Approach, *Econometrica* 59(2), 347–70