

# JOHANSEN COINTEGRATION TEST IN EVIEWS 8 (COMPUTER LAB 2 – FURTHER NOTES)

Financial Modelling and Business Forcecasting  
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Mrs Hajar Raouf

# EvIEWS Practicals – Time Series

- **Computer Lab 1:**
  - Box-Jenkins Methodology,
  - Unit Root Tests
- **Computer Lab 2:**
  - Cointegration Tests (Engle-Granger & Johansen)
- **Computer Lab 3:**
  - ARCH/GARCH Modelling
  - Forecasting from GARCH Models
- **Computer Lab 4:**
  - Practice Review

# Johansen's Approach to Cointegration

- Consider two variables,  $X_t$  and  $Y_t$ , each of which is integrated of order 1:
- $X_t \sim I(1)$  and
- $Y_t \sim I(1)$
- It can be shown that at most there exist only one cointegrating vector.
- But how about when we consider more than 2 variables?
- Also, how to choose the dependent variable?  
Residuals vary based on which time series is set as the DV.

# Johansen's Approach to Cointegration

- If we have «  $n$  » variables, there can be up to «  $n-1$  » cointegrating vectors.
- Each of these form a long-run equilibrium relationship between the variables.
- Johansen (1988) developed a method for both:
  - ✓ Determining how many cointegrating vectors there are, and
  - ✓ Estimating all the distinct relationships.

# Johansen's Approach to Cointegration

- Cointegration refers to a linear combination of nonstationary variables,
- The Johansen test approaches the testing for cointegration by examining the number of independent linear combinations ( $r$ ) for an «  $n$  » time series variables set that yields a stationary process.

# Johansen's Approach to Cointegration

- $X_{1,t} = \alpha_1 + \delta_1 Z_{1,t} + \delta_2 Z_{2,t} + \dots + \delta_p Z_{p,t} + \varepsilon_{1,t}$
- $X_{2,t} = \alpha_2 + \varphi_1 Z_{1,t} + \varphi_2 Z_{2,t} + \dots + \varphi_p Z_{p,t} + \varepsilon_{2,t}$
- ...
- $X_{n,t} = \alpha_n + \psi_1 Z_{1,t} + \psi_2 Z_{2,t} + \dots + \psi_p Z_{p,t} + \varepsilon_{n,t}$
- The number of independent linear combinations ( $r$ ) is related to the assumed number of common non-stationary underlying process ( $p$ ) as follows:  **$p = n - r$**
- **In other terms: the rank of the matrix is the same as the number of cointegrating vectors.**
- **Note:**
- *Full rank = when all the vectors in a matrix are linearly independent = all the series in the cointegrating space should be stationary (i.e  $I(0)$ ), i.e the variables do not have unit roots).*

# Johansen's Approach to Cointegration

- The number of independent linear combinations ( $r$ ) is related to the assumed number of common non-stationary underlying process ( $p$ ) as follows:

$$p = n - r$$

- Three possible outcomes:
  - 1)  $r=0$ ,  $p=n$  : in this case, time series variables are not cointegrated.
  - 2)  $0 < r < n$ ,  $1 < p < n$ : in this case, the time series are cointegrated.
  - 3)  $r=n$ ,  $p=0$ , all time series are stationary ( $I(0)$ ) to start with; Cointegrating is not relevant here.

# An illustration of the Johansen's Approach to Cointegration

- By examining the number of independent combinations, we are indirectly examining the cointegration existence hypothesis.
- The Johansen test is performed through two statistics:
- **The Trace Test ( $\lambda_{\text{trace}}$ ) and,**
- **Maximum Eigenvalue ( $\lambda_{\text{max}}$ ) .**



# The Trace Test Statistic $\lambda_{\text{trace}}$

- The trace test examines the number of linear combinations ( $r$ ) to be equal to a given value ( $r_0$ ) and the alternative hypothesis for «  $r$  » to be greater than  $r_0$  :
  - $H_0 : r = r_0$
  - $H_1 : r > r_0$
- To test for the existence of cointegration using the trace test, we set  $r_0 = 0$  (**No Cointegration**), and examine whether  $H_0$  can be rejected (i.e **there is at least one cointegration relationship**).

# The Trace Test Statistic $\lambda_{\text{trace}}$

- $H_0 : r = r_0$
- $H_1 : r > r_0$
- We proceed sequentially from  $r=0$  to  $r = n-1$
- Compare  $\lambda_{\text{trace}}$  to 5% critical value:
- if  $\lambda_{\text{trace}}$  is  $>$  the 5% Critical Value  $\rightarrow$  Reject  $H_0$

Null Hypothesis	Alternative Hypothesis	$\lambda_{\text{trace}}$ Value	5% Critical Value	10% Critical Value
$r=0$	$r > 0$	$\lambda_{\text{trace}}(0)$		
$r \leq 1$	$r > 1$	$\lambda_{\text{trace}}(1)$		
$r \leq 2$	$r > 2$	$\lambda_{\text{trace}}(2)$		

Not advised to reject  $H_0$  based on the 10% Critical value as using wide confidence intervals is a danger that leads to fail to reject an incorrect  $H_0$

# The Trace Test Statistic $\lambda_{\text{trace}}$

- $H_0 : r = r_0$
- $H_1 : r > r_0$

- Compare  $\lambda_{\text{trace}}$  to 5% critical value:
- if  $\lambda_{\text{trace}}$  is  $>$  the 5% Critical Value  $\rightarrow$  Reject  $H_0$

## Possible outcomes:

- $\lambda_{\text{trace}(0)} > 5\% \text{ Cv} : \text{One or more cointegrating vectors}$
- $\lambda_{\text{trace}(1)} > 5\% \text{ Cv} : \text{Two or three cointegrating vectors}$
- $\lambda_{\text{trace}(2)} > 5\% \text{ Cv} : \text{More than two cointegrating vectors}$

# The Max Eigenvalue Test Statistic $\lambda_{\max}$

- The maximum Eigenvalue follows the same reasoning and asks the same question yet assumes different null and alternative hypotheses:
  - $H_0 : r = r_0$
  - $H_1 : r = r_0 + 1$
- So, starting with  $r_0 = 0$  (No Cointegration), rejecting  $H_0$  means there is only one cointegration relationship (i.e: only one possible combination of non-stationary variables that generates a stationary process).

# The Max Eigenvalue Test Statistic $\lambda_{\max}$

- $H_0 : r = r_0$
- $H_1 : r = r_0 + 1$
- Compare  $\lambda_{\max}$  to 5% critical value:
- if  $\lambda_{\max}$  is  $>$  the 5% Critical Value  $\rightarrow$  Reject  $H_0$

Null Hypothesis	Alternative Hypothesis	$\lambda_{\max}$ Value	5% Critical Value	10% Critical Value
$r = 0$	$r = 1$	$\lambda_{\max(0,1)}$		
$r = 1$	$r = 2$	$\lambda_{\max(1,2)}$		
$r = 2$	$r = 3$	$\lambda_{\max(2,3)}$		

Not advised to reject  $H_0$  based on the 10% Critical value as using wide confidence intervals is a danger that leads to fail to reject an incorrect  $H_0$

# The MaxTest Statistic $\lambda_{\text{trace}}$

- $H_0 : r = r_0$
- $H_1 : r = r_0 + 1$
- Compare  $\lambda_{\text{trace}}$  to 5% critical value:
- if  $\lambda_{\text{trace}}$  is  $>$  the 5% Critical Value  $\rightarrow$  Reject  $H_0$

## Possible outcomes:

- $\lambda_{\max(0,1)} > 5\% \text{ Cv} : \text{Only one cointegrating vector}$
- $\lambda_{\max(1,2)} > 5\% \text{ Cv} : \text{Two cointegrating vectors}$
- $\lambda_{\max(2,3)} > 5\% \text{ Cv} : \text{Three cointegrating vectors}$

# Johansen Cointegration Test in Eviews 8

## Steps:

- Open all variables as « Group » then:
- Quick > Group Stat. > Johansen Cointegration Test > Type all the variables under study (all of them should be at level) > Click OK.
- Choose Option « 3 » in the dialog box (! : Number of lags)
- What now? Based on results obtained, when rejecting the null hypothesis (hence finding the number of cointegrated variables, i.e the ones that exhibit a long run relationship), run VECM.

# Johansen Cointegration Test Output (page 10 of handout – Practical 2)

Sample: 1981M01 1996M06  
Included observations: 182  
Series: LNX LNIT LNFR  
Lags interval: 1 to 3

Selected (0.05 level\*) Number of Cointegrating Relations by Model

Data Trend:	None	None	Linear	Linear	Quadratic
Test Type	No Intercept	Intercept	Intercept	Intercept	Intercept
	No Trend	No Trend	No Trend	Trend	Trend
Trace	1	2	1	2	3
Max-Eig	1	2	1	2	3

\*Critical values based on MacKinnon-Haug-Michelis (1999)

Information Criteria by Rank and Model

Data Trend:	None	None	Linear	Linear	Quadratic
Rank or	No Intercept	Intercept	Intercept	Intercept	Intercept
No. of CEs	No Trend	No Trend	No Trend	Trend	Trend
Log Likelihood by Rank (rows) and Model (columns)					
0	2151.201	2151.201	2158.361	2158.361	2162.674
1	2165.124	2173.629	2179.877	2180.454	2180.865
2	2169.303	2182.431	2184.155	2195.286	2195.318
3	2169.550	2184.900	2184.900	2197.460	2197.460
Akaike Information Criteria by Rank (rows) and Model (columns)					
0	-23.34287	-23.34287	-23.38858	-23.38858	-23.40301
1	-23.42993	-23.51240	-23.55909	-23.55444	-23.53697
2	-23.40993	-23.53221	-23.54017	-23.64051*	-23.62987
3	-23.34671	-23.48242	-23.48242	-23.58747	-23.58747
Schwarz Criteria by Rank (rows) and Model (columns)					
0	-22.86755	-22.86755	-22.86045	-22.86045	-22.82207
1	-22.84898	-22.91385	-22.92533*	-22.90308	-22.85040
2	-22.72336	-22.81043	-22.80078	-22.86591	-22.83767
3	-22.55451	-22.63741	-22.63741	-22.68964	-22.68964

- **Cointegration Vectors**
- Full rank = when “r” (number of cointegrating relations) = “n” (number of series), i.e. that all the vectors in the matrix are linearly independent, which also means that all the series in the cointegrating space should be I(0)



# Johansen Cointegration Test Output (page 12 of handout – Practical 2)

Sample (adjusted): 1981M06 1996M06  
Included observations: 181 after adjustments  
Trend assumption: Linear deterministic trend  
Series: LNX LNIT LNFR  
Lags interval (in first differences): 1 to 4

## Unrestricted Cointegration Rank Test (Trace)

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	0.05 Critical Value	Prob.**
None *	0.209924	51.34325	29.79707	0.0001
At most 1	0.040115	8.694890	15.49471	0.3944
At most 2	0.007071	1.284418	3.841466	0.2571

number of cointegrating relations  
under the null hypothesis

Results to compare with 5%  
Critical values

Trace test indicates 1 cointegrating eqn(s) at the 0.05 level

\* denotes rejection of the hypothesis at the 0.05 level

\*\*Mackinnon-Haug-Michelis (1999) p-values

## Unrestricted Cointegration Rank Test (Maximum Eigenvalue)

Hypothesized No. of CE(s)	Eigenvalue	Max-Eigen Statistic	0.05 Critical Value	Prob.**
None *	0.209924	42.64836	21.13162	0.0000
At most 1	0.040115	7.410473	14.26460	0.4419
At most 2	0.007071	1.284418	3.841466	0.2571

Results to compare with 5% Critical values

Max-eigenvalue test indicates 1 cointegrating eqn(s) at the 0.05 level

\* denotes rejection of the hypothesis at the 0.05 level

\*\*Mackinnon-Haug-Michelis (1999) p-values

## Unrestricted Cointegrating Coefficients (normalized by b'S11\*b=I):

LNK	LNIT	LNFR
9.441996	-66.44157	122.7390
-15.59406	51.65654	-72.59837
-4.999839	-17.42788	29.89347

## Unrestricted Adjustment Coefficients (alpha):

LNK	LNIT	LNFR
-0.002189	0.002510	0.001154

## Note that:

- The trace statistic and the maximum eigenvalue statistic may yield conflicting results. For such cases, you need to examine the estimated cointegrating vector and base your choice on the interpretability of the cointegrating relations; see Johansen and Juselius (1990) for an example.

# VECM in Eviews 8

## Steps to examine the entire VECM:

- In the « Johansen Cointegration Test » window – Proc > Make Vector Autoregression > Click OK.
- Fill the dialog box « VAR Specification » accordingly,
- VECM output we get a system of equation (in Lab 2 practical, we get 3 equations  $D(LNX)$ ,  $D(LNIT)$  and  $D(LNFR)$ ).

## VECM OUTPUT

## Vector Error Correction Estimates

Date: 02/29/10 Time: 22:49

Sample (adjusted): 1981M05 1996M06

Included observations: 182 after adjustments

Standard errors in ( ) &amp; t-statistics in [ ]

Cointegrating Eq:	
CointEq1	
LN <sub>X</sub> (-1)	1.000000
LN <sub>IT</sub> (-1)	-7.323725 (0.91587) [-7.99650]
LN <sub>FR</sub> (-1)	13.77825 (1.72219) [8.00041]
C	-34.95677
Error Correction:	
	D(LN <sub>X</sub> )    D(LN <sub>IT</sub> )    D(LN <sub>FR</sub> )
CointEq1	-0.017838 (0.01081) [-1.64947]    -0.004262 (0.00150) [-2.84163]    -0.006520 (0.00102) [-6.39890]
D(LN <sub>X</sub> (-1))	0.025798 (0.07696) [0.33520]    -0.001177 (0.01067) [-0.11023]    0.005631 (0.00725) [0.77659]
D(LN <sub>X</sub> (-2))	-0.060404 (0.07525) [-0.80273]    0.008394 (0.01044) [0.80438]    0.011538 (0.00709) [1.62733]
D(LN <sub>X</sub> (-3))	0.090001 (0.07574) [1.18824]    0.001448 (0.01050) [0.13784]    -0.003868 (0.00714) [-0.54203]
D(LN <sub>IT</sub> (-1))	0.536944 (0.57352) [0.93622]    0.284263 (0.07954) [3.57395]    -0.132223 (0.05404) [-2.44691]
D(LN <sub>IT</sub> (-2))	-1.231853 (0.60718) [-2.02883]    -0.005679 (0.08420) [-0.06745]    0.085820 (0.05721) [1.50016]

A system of equation of  
3 Dependent Variables

About 1.7% of disequilibrium  
is corrected each month by  
the changes in LN<sub>X</sub>

# VECM in Eviews 8

## Steps to examine the entire VECM (follow-up):

- We need the p-values for the CointEq1. How to do it?

**Proc > Make System > Order by variable (we obtain the full system of equations),**

- Copy the first equation, go to Quick > Estimate Equation > Paste the equation in the dialog box > Ok.

The screenshot shows the EViews 'System' window with the title bar 'System: UNTITLED Workfile: PPP::Ppp\'. The window contains three equations:

**D(LNX) =** C(1)\*( LNX(-1) - 7.32372487543\*LNIT(-1) + 13.7782465024\*LNFR(-1) - 34.9567692797 ) + C(2)\*D(LNX(-1)) + C(3)\*D(LNX(-2)) + C(4)\*D(LNX(-3)) + C(5)\*D(LNIT(-1)) + C(6)\*D(LNIT(-2)) + C(7)\*D(LNIT(-3)) + C(8)\*D(LNFR(-1)) + C(9)\*D(LNFR(-2)) + C(10)\*D(LNFR(-3)) + C(11)

**D(LNIT) =** C(12)\*( LNX(-1) - 7.32372487543\*LNIT(-1) + 13.7782465024\*LNFR(-1) - 34.9567692797 ) + C(13)\*D(LNX(-1)) + C(14)\*D(LNX(-2)) + C(15)\*D(LNX(-3)) + C(16)\*D(LNIT(-1)) + C(17)\*D(LNIT(-2)) + C(18)\*D(LNIT(-3)) + C(19)\*D(LNFR(-1)) + C(20)\*D(LNFR(-2)) + C(21)\*D(LNFR(-3)) + C(22)

**D(LNFR) =** C(23)\*( LNX(-1) - 7.32372487543\*LNIT(-1) + 13.7782465024\*LNFR(-1) - 34.9567692797 ) + C(24)\*D(LNX(-1)) + C(25)\*D(LNX(-2)) + C(26)\*D(LNX(-3)) + C(27)\*D(LNIT(-1)) + C(28)\*D(LNIT(-2)) + C(29)\*D(LNIT(-3)) + C(30)\*D(LNFR(-1)) + C(31)\*D(LNFR(-2)) + C(32)\*D(LNFR(-3)) + C(33)

In the first equation, the coefficient C(1) is circled in red. In the third equation, the coefficient C(33) is circled in red.

View Proc Object Print Name Freeze Estimate Forecast Stats Resids

Dependent Variable: D(LNX)

Method: Least Squares

Date: 02/29/16 Time: 23:11

Sample (adjusted): 1981M05 1996M06

Included observations: 182 after adjustments

$$D(LNX) = C(1) * (LN X(-1) - 7.32372487543 * LNIT(-1) + 13.7782465024 \\ * LNFR(-1) - 34.9567692797) + C(2) * D(LNX(-1)) + C(3) * D(LNX(-2)) + \\ C(4) * D(LNX(-3)) + C(5) * D(LNIT(-1)) + C(6) * D(LNIT(-2)) + C(7) * D(LNIT(-3)) + C(8) * D(LNFR(-1)) + C(9) * D(LNFR(-2)) + C(10) * D(LNFR(-3)) + C(11)$$

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	-0.017838	0.010814	-1.649470	0.1009
C(2)	0.025798	0.076963	0.335199	0.7379
C(3)	-0.060404	0.075249	-0.802727	0.4232
C(4)	0.090001	0.075744	1.188237	0.2364
C(5)	0.536944	0.573525	0.936217	0.3505
C(6)	-1.231853	0.607175	-2.028826	0.0440
C(7)	0.140394	0.602074	0.233184	0.8159
C(8)	-0.520290	0.760372	-0.684257	0.4947
C(9)	0.023461	0.787664	0.029785	0.9763
C(10)	-1.702787	0.725115	-2.348298	0.0200
C(11)	0.013197	0.006035	2.186572	0.0301

R-squared	0.084431	Mean dependent var	0.001811
Adjusted R-squared	0.030889	S.D. dependent var	0.020025
S.E. of regression	0.019713	Akaike info criterion	-4.956558
Sum squared resid	0.066450	Schwarz criterion	-4.762909
Log likelihood	462.0468	Hannan-Quinn criter.	-4.878056
F-statistic	1.576910	Durbin-Watson stat	1.985281
Prob(F-statistic)	0.117086		

# VECM in Eviews 8

## Steps to examine the entire VECM (follow-up):

- The output will show results for the first difference of variables in the system of equation,
- !!!: In the output, you need to find a negative sign and significant first coefficient (in cointeq1): This means that if there is any drift from long run relationship, there is an error correction mechanism that brings back the variable to the long run equilibrium.

# VECM in Eviews 8

## Steps to examine the entire VECM (follow-up):

- What if this first coefficient is negative but insignificant?
- This is an indication that the model specification probably needs readjustment and / or the problem lies in the data (you need to have enough observations compared to the number of parameters you want to estimate).



# Further References

- Johansen, Søren (1991). "Estimation and Hypothesis Testing of Cointegration Vectors in Gaussian Vector Autoregressive Models". *Econometrica* 59 (6): 1551–1580.
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- Brooks, Chris (2014), "Introductory Econometrics in Finance", 3rd edition, Cambridge University Press.