ARCH & GARCH MODELS

Dzulfian Syafrian

Feb-Mar 2018/19

Dataset

- Download and open garch.xls as well as FMBF Practical 3.pdf file
- Dataset: Change ‘dy’ to ‘jy’

Generate new return variables

Quick > generate series > type:

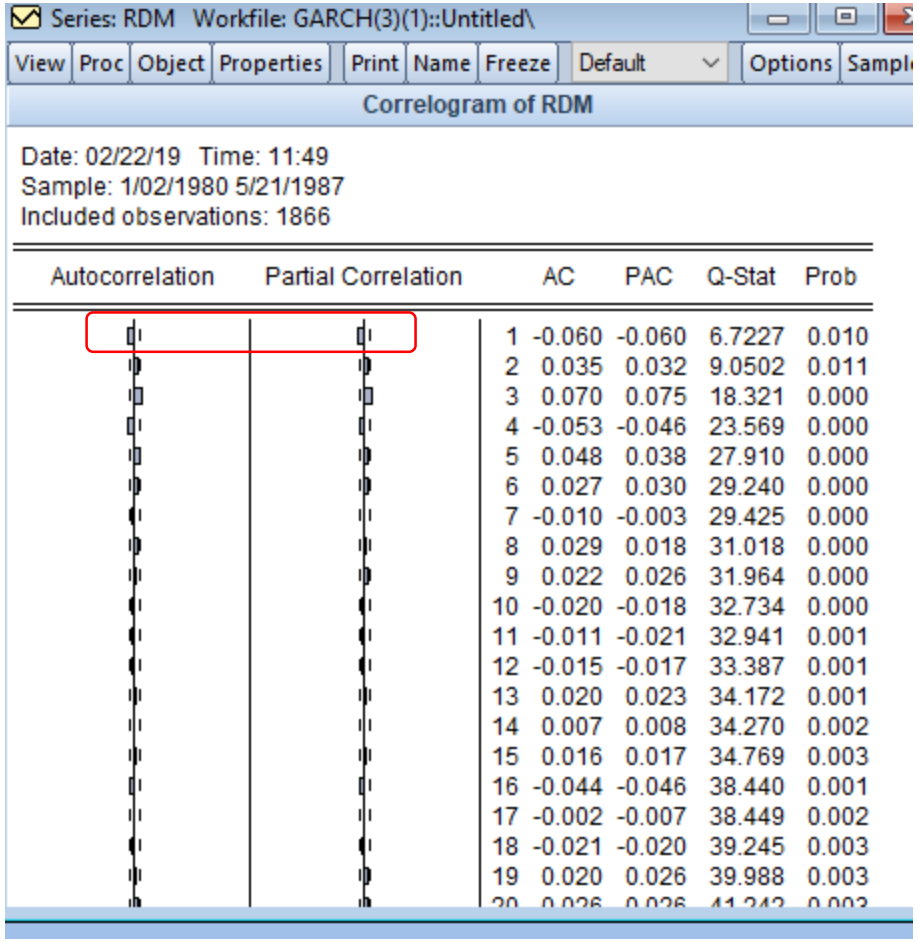
- rbp=dlog(bp)
- rcd=dlog(cd)
- rdm=dlog(dm)
- rjy=dlog(jy)
- rsf=dlog(sf)

Produce figures

- click rdm > right click > open > view > graph > ok
- Check other variables

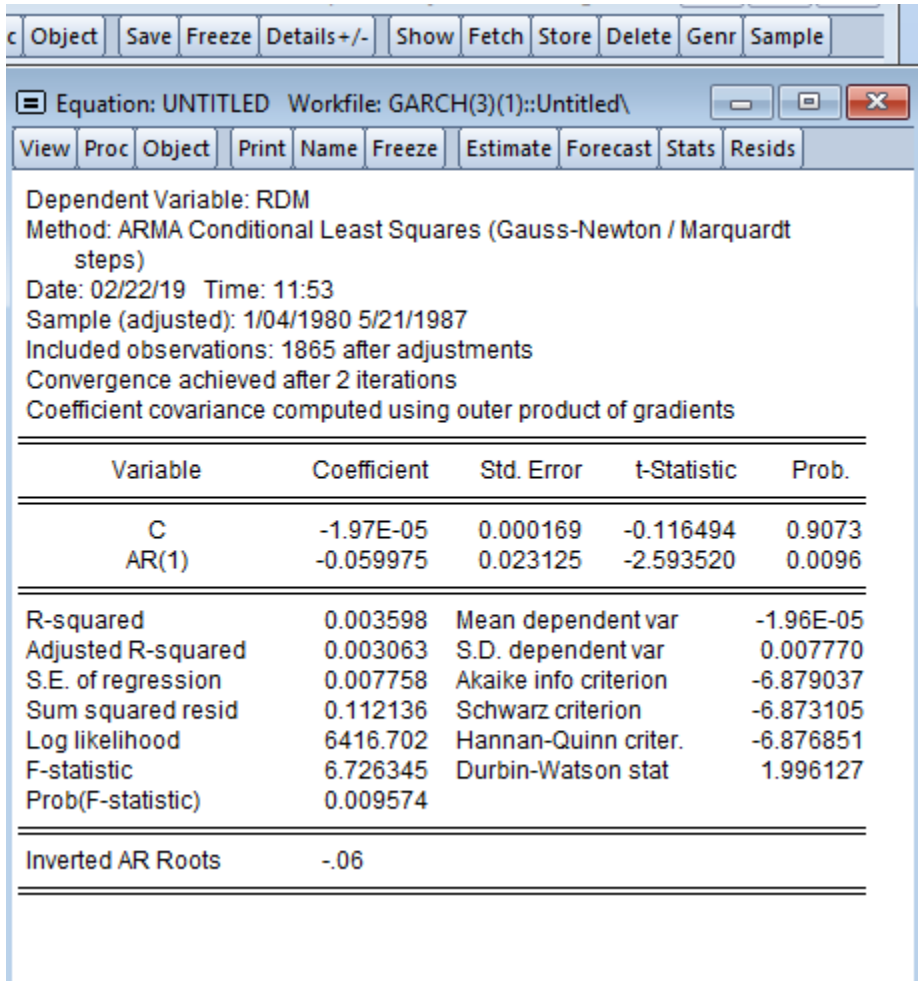
Identify ARMA

- quick > series stat > correlogram > rdm > conclusion: arma (1,1)

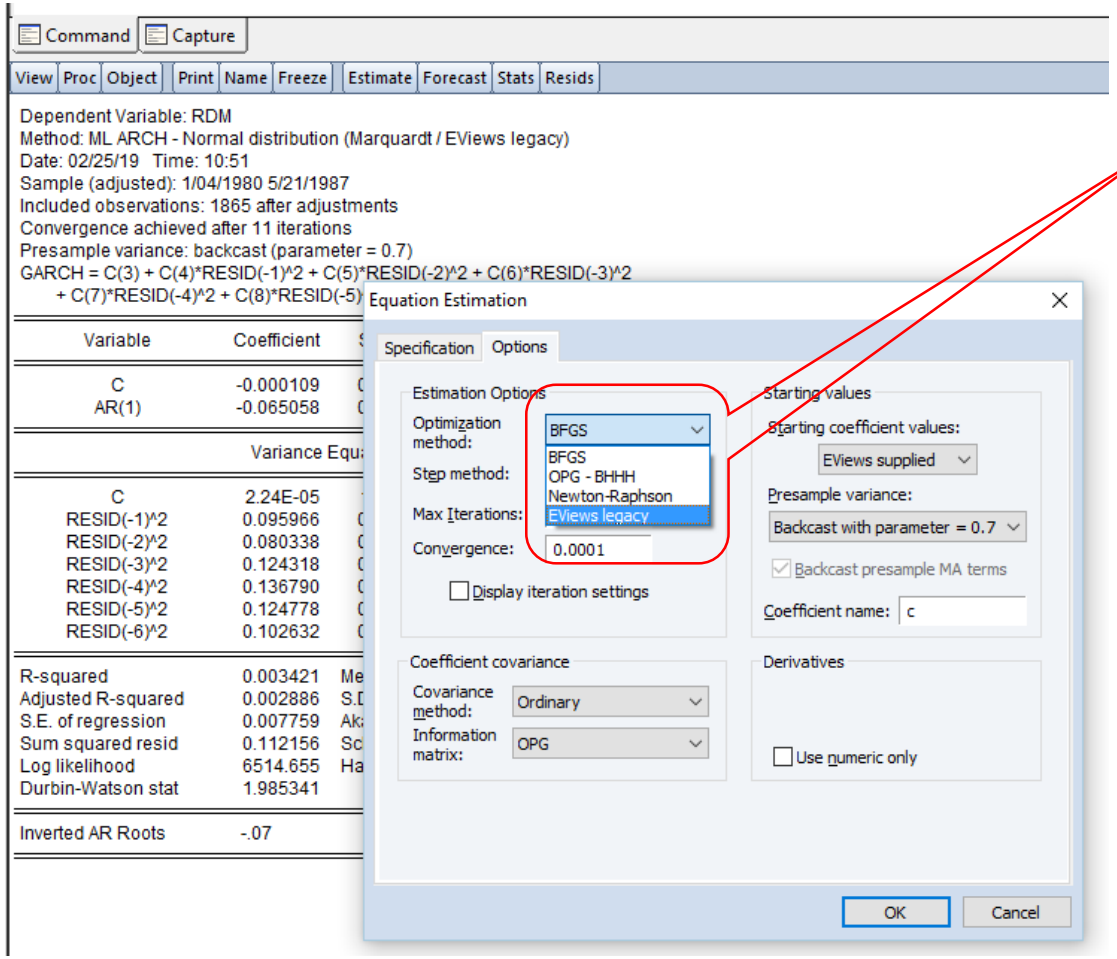


Run AR(1)

- quick > estimate equation > rdm c ar(1) > options > cls > ok



Change the optimisation methods



- From BFGS to Eviews Legacy
- To get the same results with the handout

ARCH Effects

Heteroskedasticity Test: ARCH

F-statistic	13.76245	Prob. F(6,1852)	0.0000
Obs*R-squared	79.34891	Prob. Chi-Square(6)	0.0000

Test Equation:
Dependent Variable: RESID^2
Method: Least Squares
Date: 02/25/19 Time: 09:28
Sample (adjusted): 1/14/1980 5/21/1987
Included observations: 1859 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	3.63E-05	4.01E-06	9.043649	0.0000
RESID^2(-1)	0.072848	0.023044	3.161237	0.0016
RESID^2(-2)	0.051576	0.023058	2.236758	0.0254
RESID^2(-3)	0.038979	0.023068	1.689751	0.0912
RESID^2(-4)	0.042468	0.023068	1.840974	0.0658
RESID^2(-5)	0.064232	0.023058	2.785648	0.0054
RESID^2(-6)	0.128593	0.023043	5.580464	0.0000

R-squared	0.042684	Mean dependent var	6.03E-05
Adjusted R-squared	0.039582	S.D. dependent var	0.000126
S.E. of regression	0.000123	Akaike info criterion	-15.16421
Sum squared resid	2.80E-05	Schwarz criterion	-15.14339
Log likelihood	14102.13	Hannan-Quinn criter.	-15.15654
F-statistic	13.76245	Durbin-Watson stat	2.004508
Prob(F-statistic)	0.000000		

H0: no arch effect, $\alpha_i = 0$

H1: there is arch effect, $\alpha_i \neq 0$

- if p-value < 5%
  - = reject H0 = there is arch effect

ARCH (q)

$$y_t = \beta_1 + \beta_2 x_{2t} + \ldots + \beta_k x_{kt} + u_t, \quad u_t \sim N(0, \sigma_t^2)$$

Where

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \alpha_2 u_{t-2}^2 + \ldots + \alpha_q u_{t-q}^2$$

In this case, q (lags)=6

Weakness of ARCH

ARCH (q)

$y_t = \beta_1 + \beta_2 x_{2t} + \ldots + \beta_k x_{kt} + u_t, \; u_t \sim N(0, \sigma_t^2)$

Where $\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \alpha_2 u_{t-2}^2 + \ldots + \alpha_q u_{t-q}^2$

- How many q (lags) do we need? Might be very large
- Consequently, the variance could be negative which means it would violate the non-negativity constraints.
- Solution: GARCH (1,1): or in general GARCH (p,q)

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta \sigma_{t-1}^2$$

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i u_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2$$

GARCH (1,1)

Dependent Variable: RDM
Method: ML ARCH - Normal distribution (Marquardt / EViews legacy)
Date: 02/25/19 Time: 10:56
Sample (adjusted): 1/04/1980 5/21/1987
Included observations: 1865 after adjustments
Convergence achieved after 10 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	-0.000222	0.000135	-1.644172	0.1001
AR(1)	-0.078108	0.025072	-3.115312	0.0018
	Variance Equation			
C	1.31E-06	3.41E-07	3.857059	0.0001
RESID(-1)^2	0.101188	0.010809	9.361857	0.0000
GARCH(-1)	0.881806	0.013051	67.56476	0.0000

R-squared	0.002478	Mean dependent var	-1.96E-05
Adjusted R-squared	0.001943	S.D. dependent var	0.007770
S.E. of regression	0.007763	Akaike info criterion	-6.998451
Sum squared resid	0.112262	Schwarz criterion	-6.983622
Log likelihood	6531.055	Hannan-Quinn criter.	-6.992987
Durbin-Watson stat	1.957207		

Inverted AR Roots	-.08

- GARCH (1,1):

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta \sigma_{t-1}^2$$

Weakness of Basic GARCH

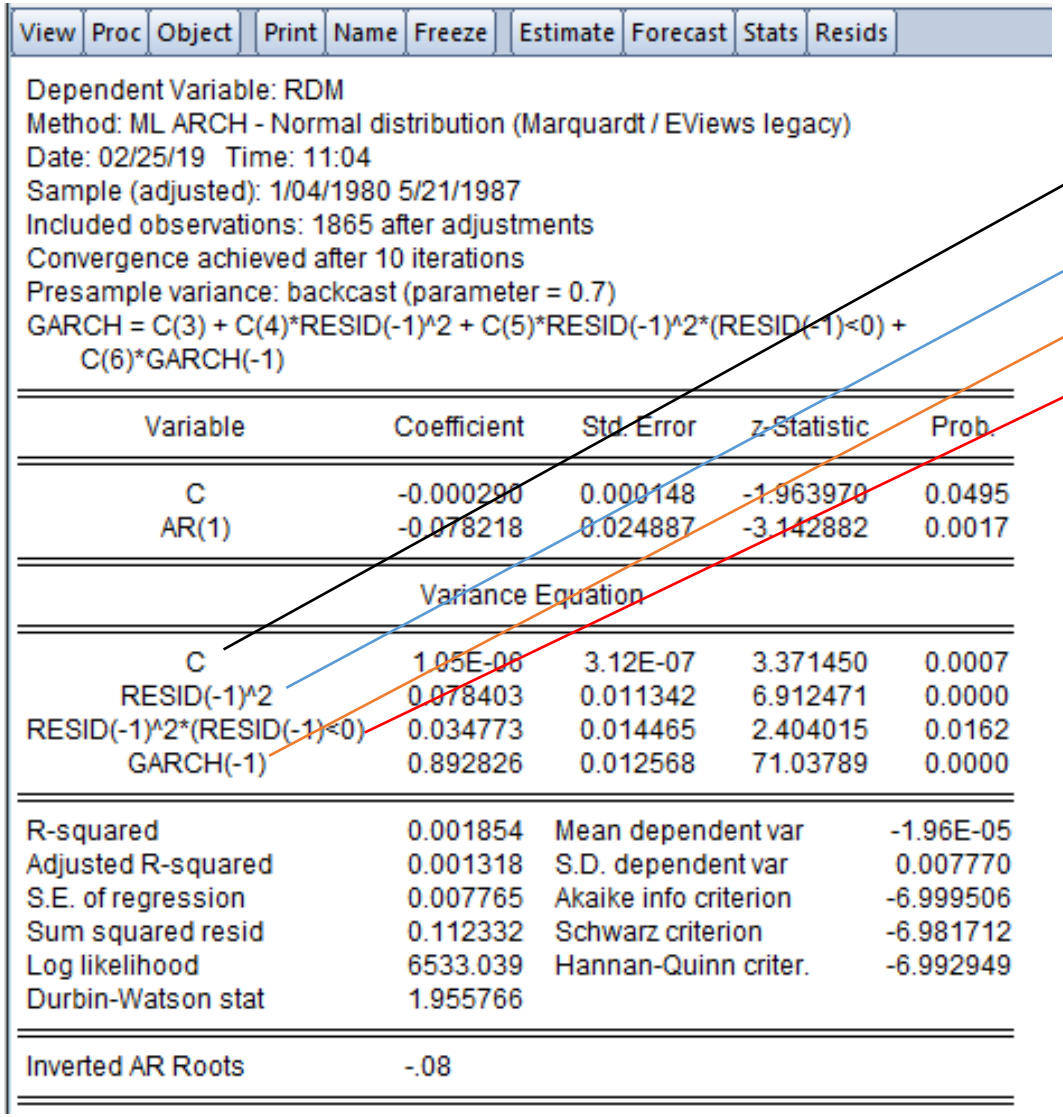
GARCH (p,q)

$y_t = \beta_1 + \beta_2 x_{2t} + \ldots + \beta_k x_{kt} + u_t, \; u_t \sim N(0, \sigma_t^2)$

Where $\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i u_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2$

- Cannot capture leverage effects: asymmetric shocks
- Solutions:
  - GJR-GARCH and E-GARCH

2. The GJR Model



View | Proc | Object | Print | Name | Freeze | Estimate | Forecast | Stats | Resids

Dependent Variable: RDM
Method: ML ARCH - Normal distribution (Marquardt / EViews legacy)
Date: 02/25/19 Time: 11:04
Sample (adjusted): 1/04/1980 5/21/1987
Included observations: 1865 after adjustments
Convergence achieved after 10 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*RESID(-1)^2*(RESID(-1)<0) + C(6)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	-0.000290	0.000148	-1.963970	0.0495
AR(1)	-0.078218	0.024887	-3.142882	0.0017

Variance Equation

C	1.05E-06	3.12E-07	3.371450	0.0007
RESID(-1)^2	0.078403	0.011342	6.912471	0.0000
RESID(-1)^2*(RESID(-1)<0)	0.034773	0.014465	2.404015	0.0162
GARCH(-1)	0.892826	0.012568	71.03789	0.0000

R-squared	0.001854	Mean dependent var	-1.96E-05
Adjusted R-squared	0.001318	S.D. dependent var	0.007770
S.E. of regression	0.007765	Akaike info criterion	-6.999506
Sum squared resid	0.112332	Schwarz criterion	-6.981712
Log likelihood	6533.039	Hannan-Quinn criter.	-6.992949
Durbin-Watson stat	1.955766		

Inverted AR Roots	-.08

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta\sigma_{t-1}^2 + \gamma u_{t-1}^2 I_{t-1}$$

where $I_{t-1} = 1$ if $u_{t-1} < 0$

$= 0$ otherwise

- For a leverage effect, we would see $\gamma > 0$.
- We require $\alpha_1 + \gamma \geq 0$ and $\alpha_1 \geq 0$ for non-negativity.

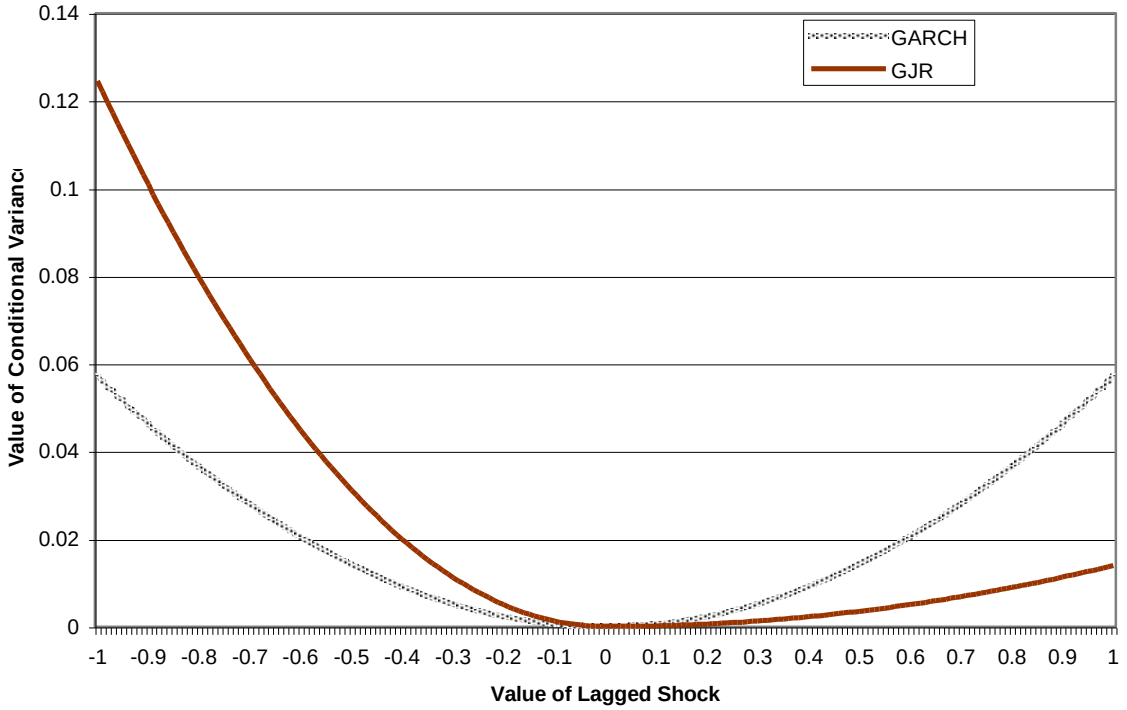
Leverage effects

- GJR-GARCH (1,1):

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta \sigma_{t-1}^2 + \gamma u_{t-1}^2 I_{t-1}$$

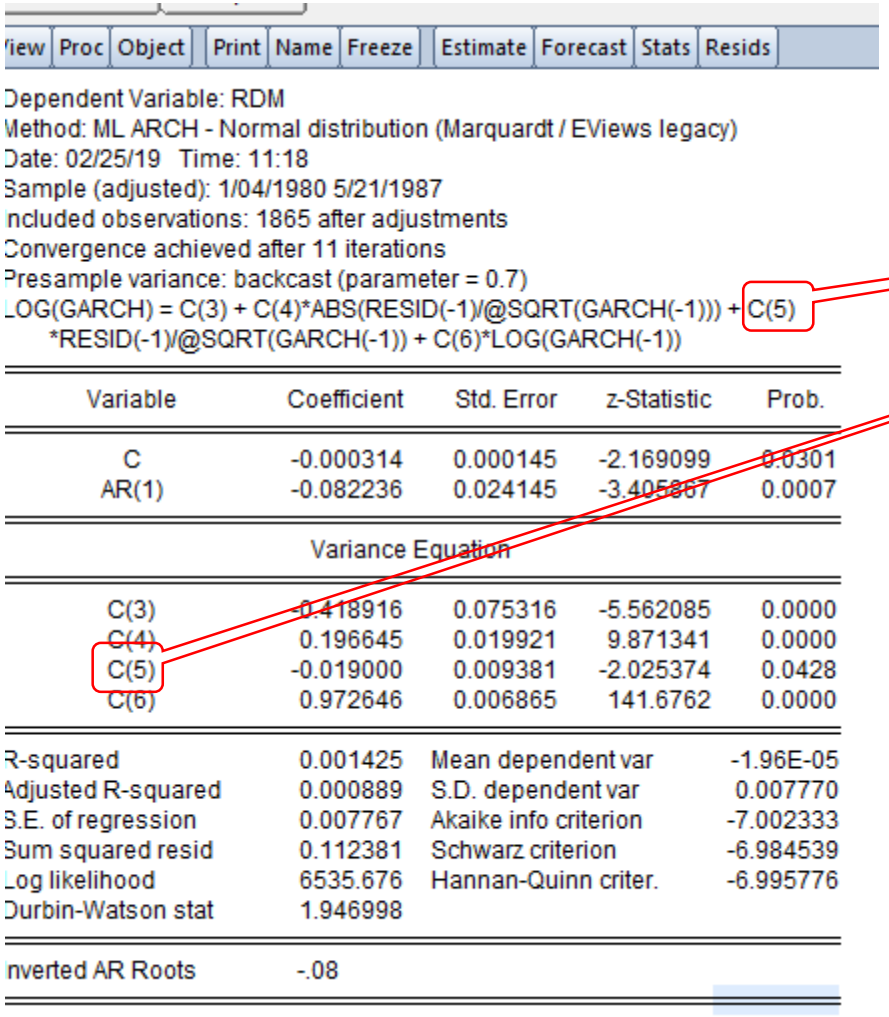
- Using monthly S&P 500 returns, Dec 1979- Jun 1998
- Estimating a GJR model, Brooks obtain the following results:

$$\sigma_t^2 = 1.243 + 0.015 u_{t-1}^2 + 0.498 \sigma_{t-1}^2 + 0.604 u_{t-1}^2 I_{t-1}$$
$$(16.372) \quad (0.437) \quad (14.999) \quad (5.772)$$



Source: Brooks, Lecture Slides: Volatility and Correlation, p.32-3

1. The EGARCH Model



View | Proc | Object | Print | Name | Freeze | Estimate | Forecast | Stats | Resids

Dependent Variable: RDM
Method: ML ARCH - Normal distribution (Marquardt / EViews legacy)
Date: 02/25/19 Time: 11:18
Sample (adjusted): 1/04/1980 5/21/1987
Included observations: 1865 after adjustments
Convergence achieved after 11 iterations
Presample variance: backcast (parameter = 0.7)
LOG(GARCH) = C(3) + C(4)*ABS(RESID(-1)/@SQRT(GARCH(-1))) + C(5)
*RESID(-1)/@SQRT(GARCH(-1)) + C(6)*LOG(GARCH(-1))

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	-0.000314	0.000145	-2.169099	0.0301
AR(1)	-0.082236	0.024145	-3.405867	0.0007
Variance Equation				
C(3)	-0.418916	0.075316	-5.562085	0.0000
C(4)	0.196645	0.019921	9.871341	0.0000
C(5)	-0.019000	0.009381	-2.025374	0.0428
C(6)	0.972646	0.006865	141.6762	0.0000

R-squared	0.001425	Mean dependent var	-1.96E-05
Adjusted R-squared	0.000889	S.D. dependent var	0.007770
S.E. of regression	0.007767	Akaike info criterion	-7.002333
Sum squared resid	0.112381	Schwarz criterion	-6.984539
Log likelihood	6535.676	Hannan-Quinn criter.	-6.995776
Durbin-Watson stat	1.946998		

Inverted AR Roots	-.08

$$\log(\sigma_t^2) = \omega + \beta \log(\sigma_{t-1}^2) + \gamma \frac{u_{t-1}}{\sqrt{\sigma_{t-1}^2}} + \alpha \left[\frac{|u_{t-1}|}{\sqrt{\sigma_{t-1}^2}} - \sqrt{\frac{2}{\pi}} \right]$$

- Because σ_t^2 is in log, its value will always be positive.
- The leverage effect is captured by γ, *where* γ <0.
  - Because if Ut-1<0 and γ<0, $\log(\sigma_t^2)$ would be higher than when Ut-1>0.
- γ shows the relationship between volatility and returns

3. GARCH-in Mean

$$y_t = \mu + \delta\sigma_{t-1} + u_t \quad , u_t \sim \mathrm{N}(0, \sigma_t^2)$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta\sigma_{t-1}^2$$

- δ = risk premium.
- 3 types GARCH-in Mean:
  1. Std. dev
  2. Variance
  3. Log(var)

3.1 GARCH-in Mean: Std. dev

$$y_t = \mu + \delta\sigma_{t-1} + u_t \quad , u_t \sim N(0, \sigma_t^2)$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta\sigma_{t-1}^2$$

Dependent Variable: RDM
Method: ML ARCH - Normal distribution (Marquardt / EViews legacy)
Date: 02/25/19 Time: 11:28
Sample (adjusted): 1/04/1980 5/21/1987
Included observations: 1865 after adjustments
Convergence achieved after 12 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(4) + C(5)*RESID(-1)^2 + C(6)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
@SQRT(GARCH)	0.244695	0.087546	2.795034	0.0052
C	-0.001847	0.000595	-3.101392	0.0019
AR(1)	-0.085665	0.025174	-3.402917	0.0007
Variance Equation				
C	1.23E-06	3.19E-07	3.865632	0.0001
RESID(-1)^2	0.098935	0.010504	9.418936	0.0000
GARCH(-1)	0.885149	0.012378	71.50928	0.0000

R-squared	0.008327	Mean dependent var	-1.96E-05
Adjusted R-squared	0.007262	S.D. dependent var	0.007770
S.E. of regression	0.007742	Akaike info criterion	-7.002372
Sum squared resid	0.111604	Schwarz criterion	-6.984578
Log likelihood	6535.712	Hannan-Quinn criter.	-6.995816
Durbin-Watson stat	1.958257		

Inverted AR Roots	-.09

- δ = risk premium.
- 3 types GARCH-in Mean:
  1. Std. dev

3.2 GARCH-in Mean: Variance

$$y_t = \mu + \delta\sigma_{t-1} + u_t \quad , u_t \sim \mathrm{N}(0, \sigma_t^2)$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta\sigma_{t-1}^2$$

Dependent Variable: RDM
Method: ML ARCH - Normal distribution (Marquardt / EViews legacy)
Date: 02/25/19 Time: 11:32
Sample (adjusted): 1/04/1980 5/21/1987
Included observations: 1865 after adjustments
Convergence achieved after 13 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(4) + C(5)*RESID(-1)^2 + C(6)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
GARCH	15.71648	5.904249	2.661892	0.0078
C	-0.000969	0.000304	-3.186633	0.0014
AR(1)	-0.086602	0.025170	-3.440729	0.0006
Variance Equation				
C	1.24E-06	3.21E-07	3.870716	0.0001
RESID(-1)^2	0.098954	0.010479	9.442854	0.0000
GARCH(-1)	0.884995	0.012385	71.45897	0.0000

R-squared	0.007702	Mean dependent var	-1.96E-05
Adjusted R-squared	0.006636	S.D. dependent var	0.007770
S.E. of regression	0.007744	Akaike info criterion	-7.002194
Sum squared resid	0.111674	Schwarz criterion	-6.984400
Log likelihood	6535.546	Hannan-Quinn criter.	-6.995638
Durbin-Watson stat	1.958761		

Inverted AR Roots	-.09

- δ = risk premium.
- 3 types GARCH-in Mean:
  2. Variance

3.3 GARCH-in Mean: Log(Var)

$$y_t = \mu + \delta\sigma_{t-1} + u_t \quad , u_t \sim \mathrm{N}(0, \sigma_t^2)$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta\sigma_{t-1}^2$$

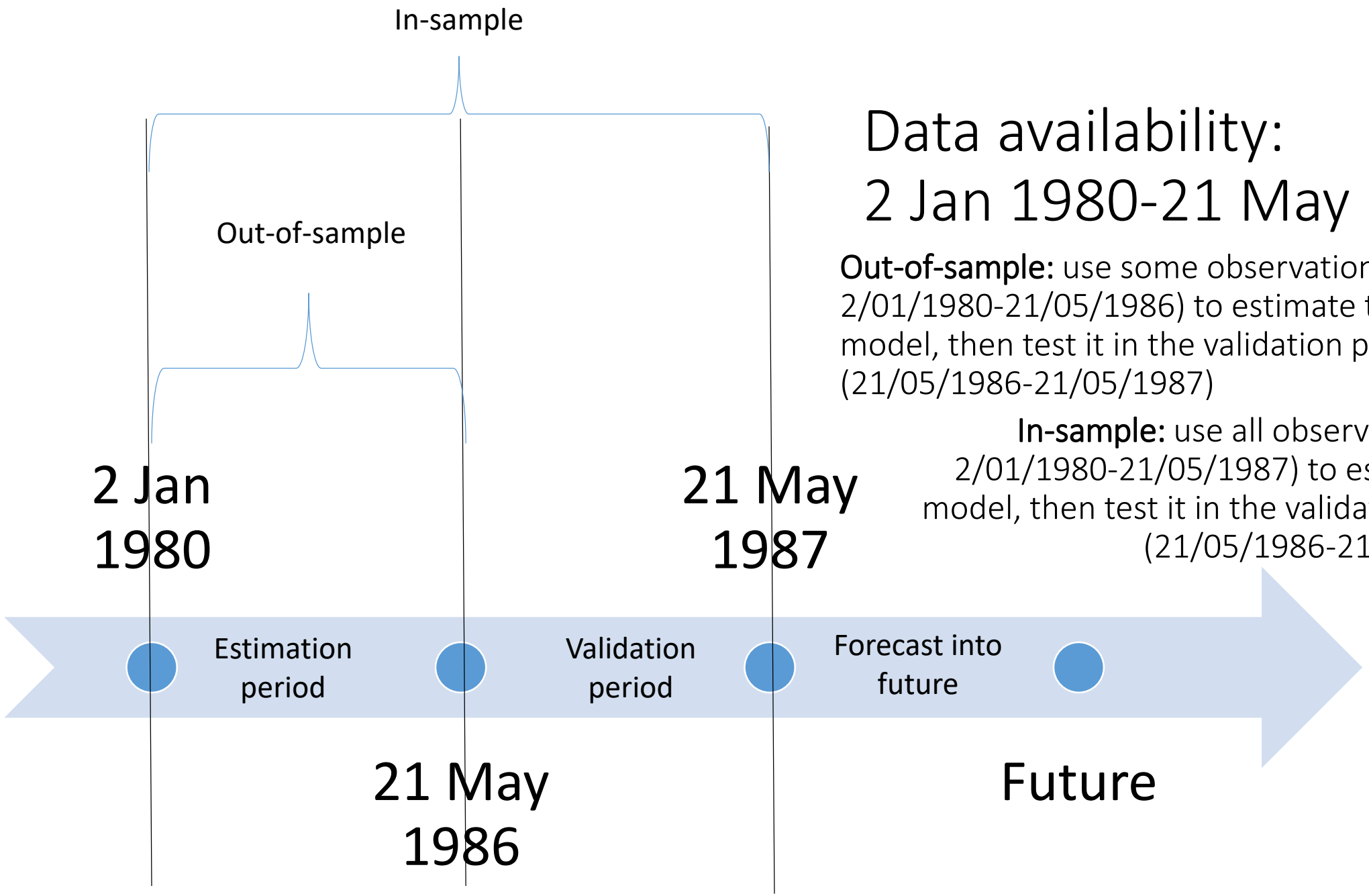
Dependent Variable: RDM
Method: ML ARCH - Normal distribution (Marquardt / EViews legacy)
Date: 02/25/19 Time: 11:34
Sample (adjusted): 1/04/1980 5/21/1987
Included observations: 1865 after adjustments
Convergence achieved after 10 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(4) + C(5)*RESID(-1)^2 + C(6)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
LOG(GARCH)	0.000821	0.000299	2.744274	0.0061
C	0.008070	0.003019	2.673102	0.0075
AR(1)	-0.085285	0.025114	-3.395921	0.0007
Variance Equation				
C	1.24E-06	3.21E-07	3.870447	0.0001
RESID(-1)^2	0.098684	0.010520	9.380760	0.0000
GARCH(-1)	0.885182	0.012436	71.17952	0.0000

R-squared	0.008179	Mean dependent var	-1.96E-05
Adjusted R-squared	0.007114	S.D. dependent var	0.007770
S.E. of regression	0.007743	Akaike info criterion	-7.002208
Sum squared resid	0.111620	Schwarz criterion	-6.984414
Log likelihood	6535.559	Hannan-Quinn criter.	-6.995652
Durbin-Watson stat	1.956490		

Inverted AR Roots	-.09

- δ = risk premium.
- 3 types GARCH-in Mean:
  3. Log(var)



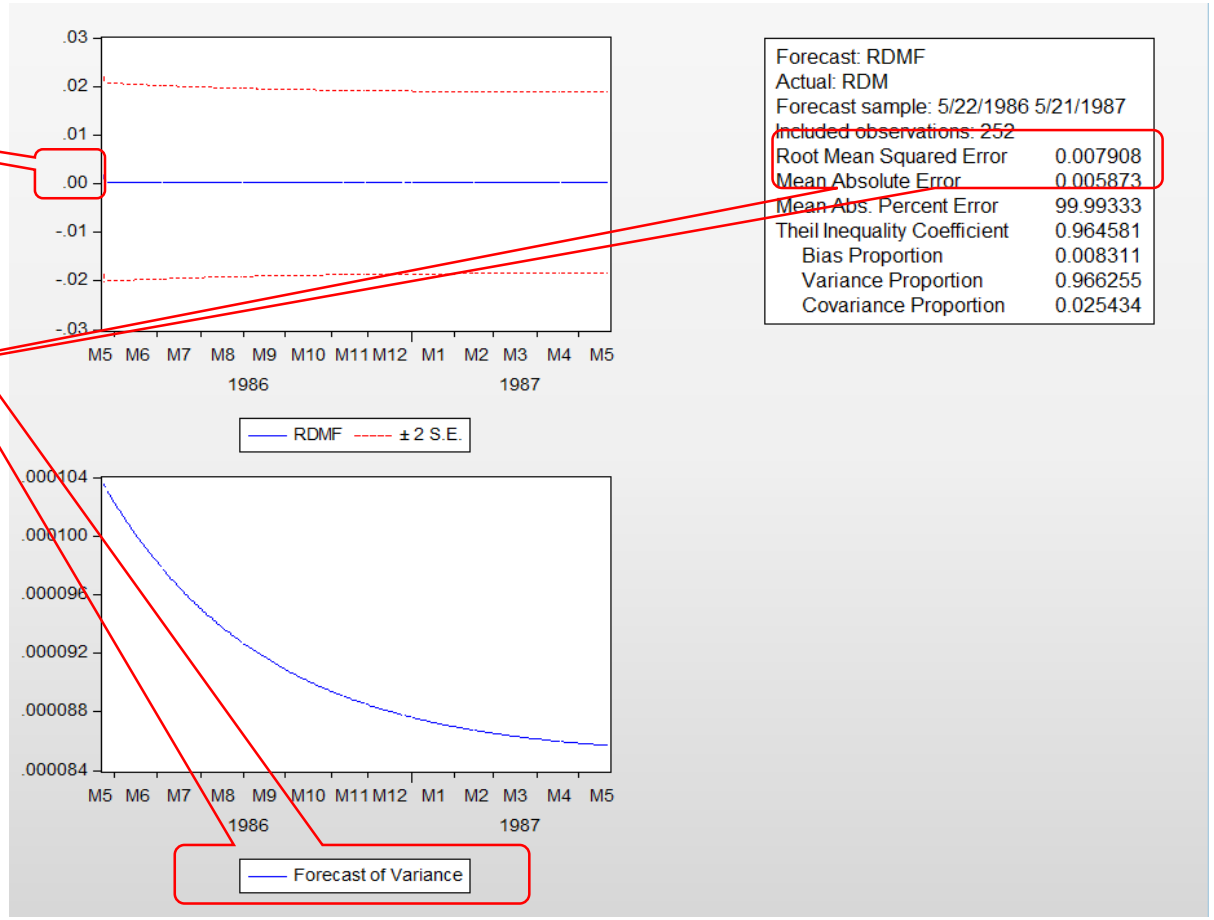
Data availability: 2 Jan 1980-21 May 1987

Out-of-sample: use some observations (e.g. 2/01/1980-21/05/1986) to estimate the model, then test it in the validation period (21/05/1986-21/05/1987)

In-sample: use all observations (e.g. 2/01/1980-21/05/1987) to estimate the model, then test it in the validation period (21/05/1986-21/05/1987).

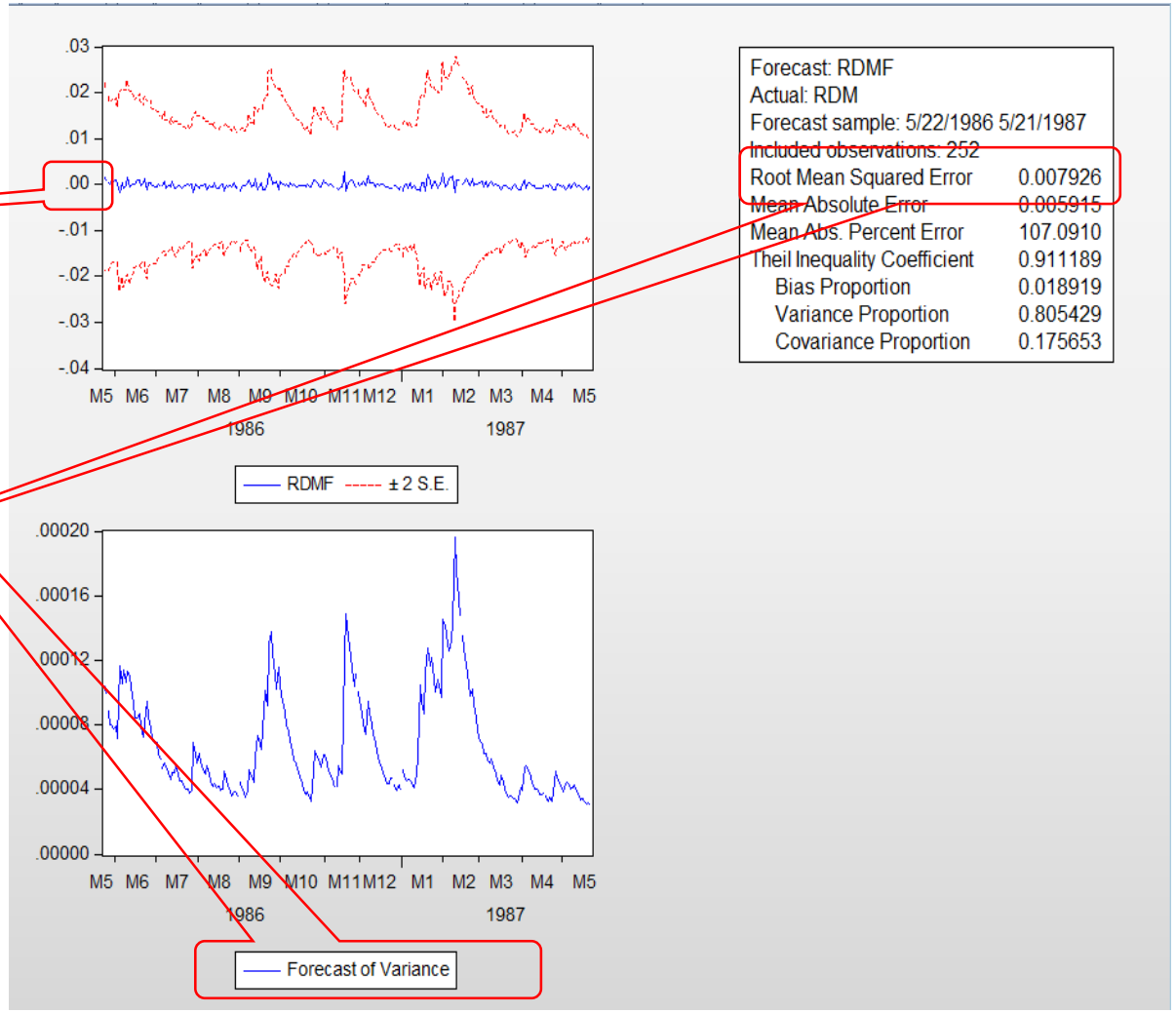
Dynamic Forecast

- The mean
- The conditional variance
  - The forecasts converge as the horizon increases
- RMSE: the difference between the actual values and forecasts, the lower-the better.



Static Forecast (rolling one-day ahead)

- The mean
- The conditional variance
- More volatility due to one-step ahead forecasts.
  - See the mean and conditional variance
- RMSE: the difference between the actual values and forecasts, the lower-the better.



Summary: (G)ARCH Models

I. ARCH: how many lags (q) do we need?
II. Basic GARCH (1,1): cannot capture the leverage effects
III. E-GARCH: allow to capture the leverage effects
IV. GJR-GARCH: allow to capture the leverage effects
V. GARCH-M (in mean): the return partly determined by its risk.
    1. Std. dev
    2. Variance
    3. Log(var)