Topic Eight: Capital Asset Pricing Model

Evelyn Lai

This lecture

› In this lecture, we will cover the following:

- Discuss a number of concepts related to selecting portfolios and fundamental to understanding the CAPM:
  - Possible portfolio risk/return outcomes
  - Optimum portfolio selection
- Derive the CAPM equation
- Examine two major CAPM issues:
  - Estimation of the CAPM parameters
  - Evaluation of the CAPM

› **Readings: BMA Chapter 8**

Portfolio Construction

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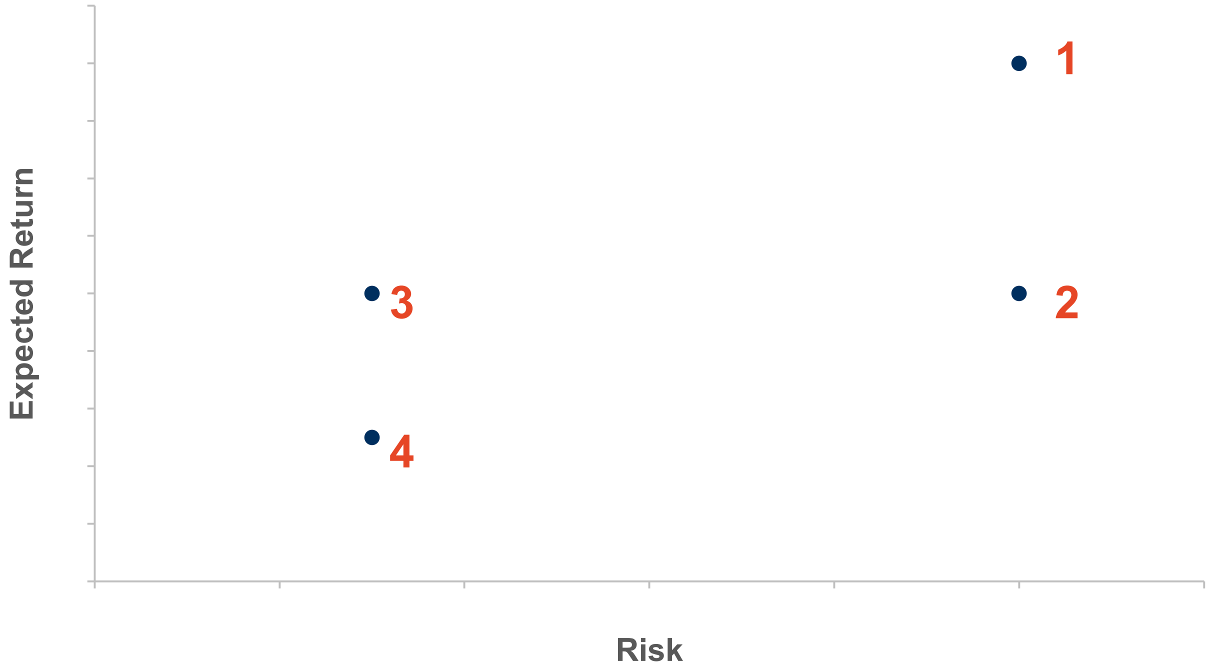
Dominance principle

› Portfolio A is preferred or dominates portfolio B if:

- $E[r_{p(A)}] \geq E[r_{p(B)}]$
- $\sigma(r_{p(A)}) \leq \sigma(r_{p(B)})$



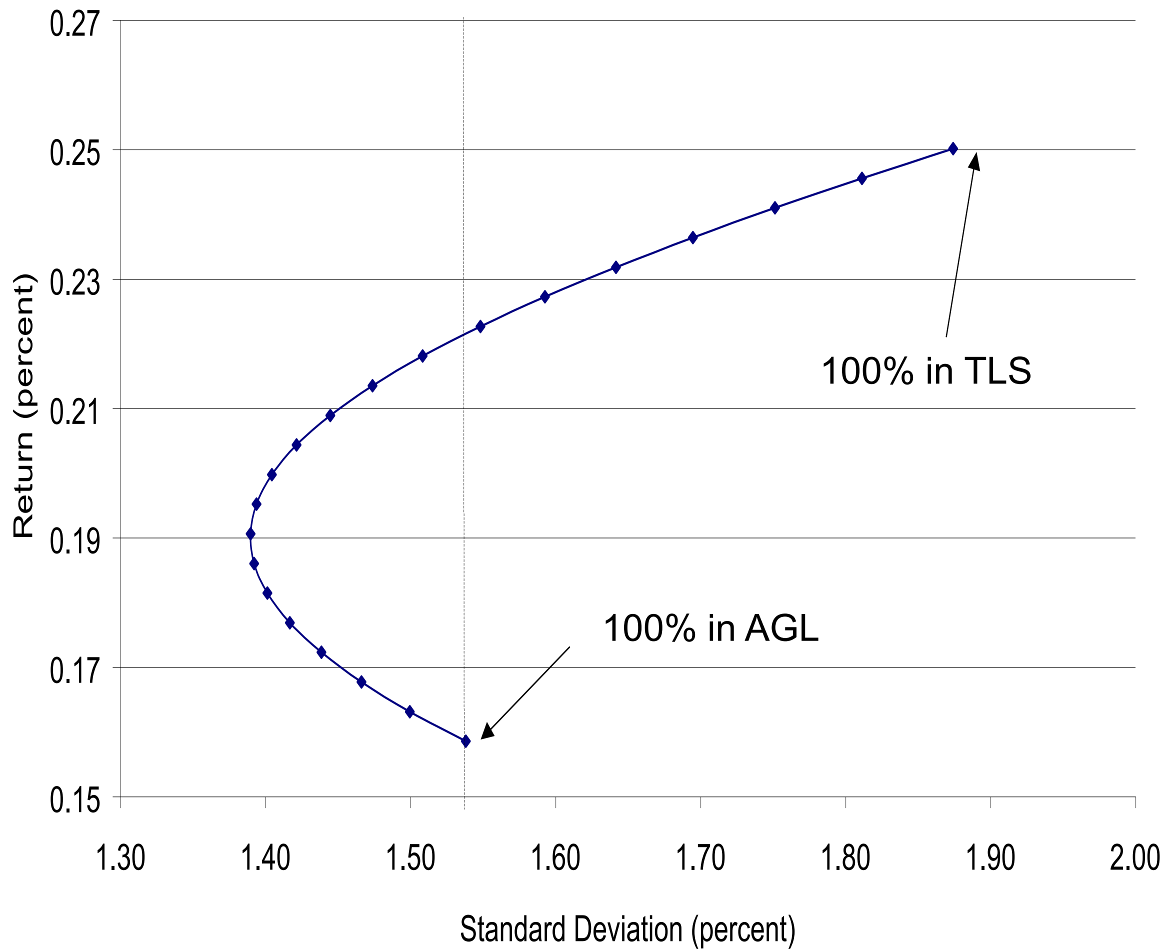
Feasible portfolios with risky assets

- The set of all return and risk outcomes that can be achieved by combinations of stocks in all possible ways are called **feasible portfolios**.

	AGL	TLS
Expected return	0.0016	0.0025
SD	0.0154	0.0187
Covariance with AGL	-	0.00011

Feasible portfolios with risky assets

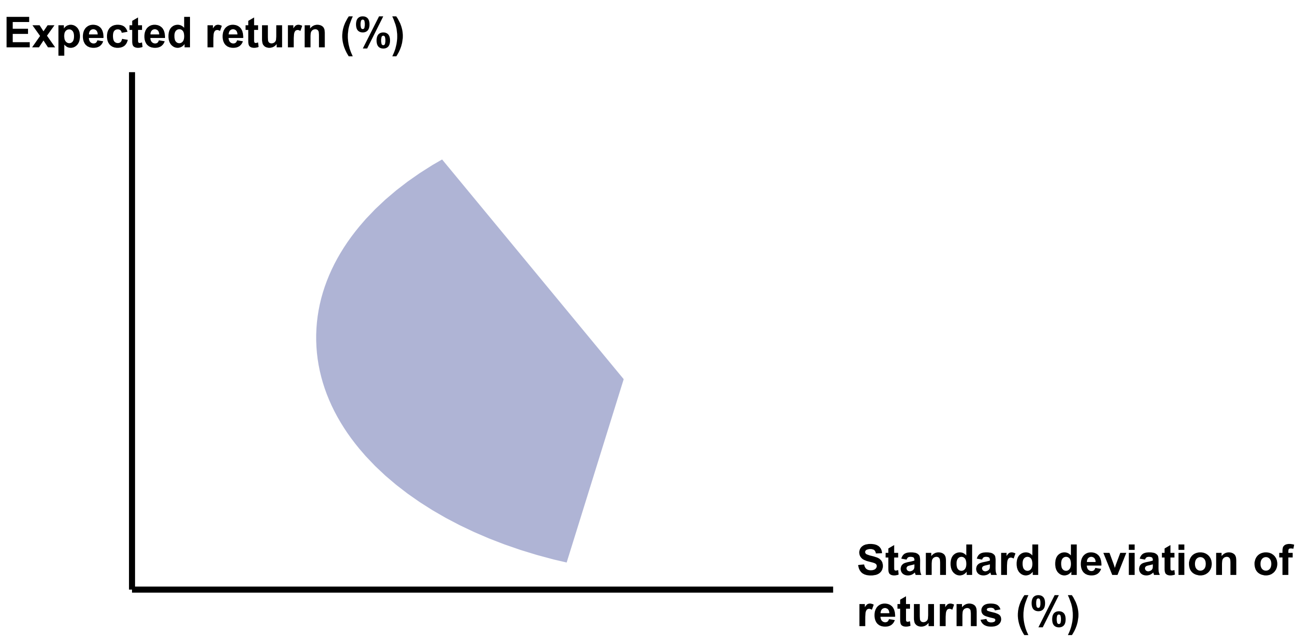
Portfolio Number	Weight in portfolio		SD (%)	Expected return (%)
	AGL	TLS	σ_p	$E(r_p)$
1	1	0	1.54	0.16
2	0.95	0.05	1.5	0.16
3	0.9	0.1	1.47	0.17
4	0.85	0.15	1.44	0.17
5	0.8	0.2	1.42	0.18
6	0.75	0.25	1.4	0.18
7	0.7	0.3	1.39	0.19
8	0.65	0.35	1.39	0.19
9	0.6	0.4	1.39	0.2
10	0.55	0.45	1.4	0.2
11	0.5	0.5	1.42	0.2
12	0.45	0.55	1.44	0.21
13	0.4	0.6	1.47	0.21
14	0.35	0.65	1.51	0.22
15	0.3	0.7	1.55	0.22
16	0.25	0.75	1.59	0.23
17	0.2	0.8	1.64	0.23
18	0.15	0.85	1.69	0.24
19	0.1	0.9	1.75	0.24
20	0.05	0.95	1.81	0.25
21	0	1	1.87	0.25



Feasible portfolios with risky assets

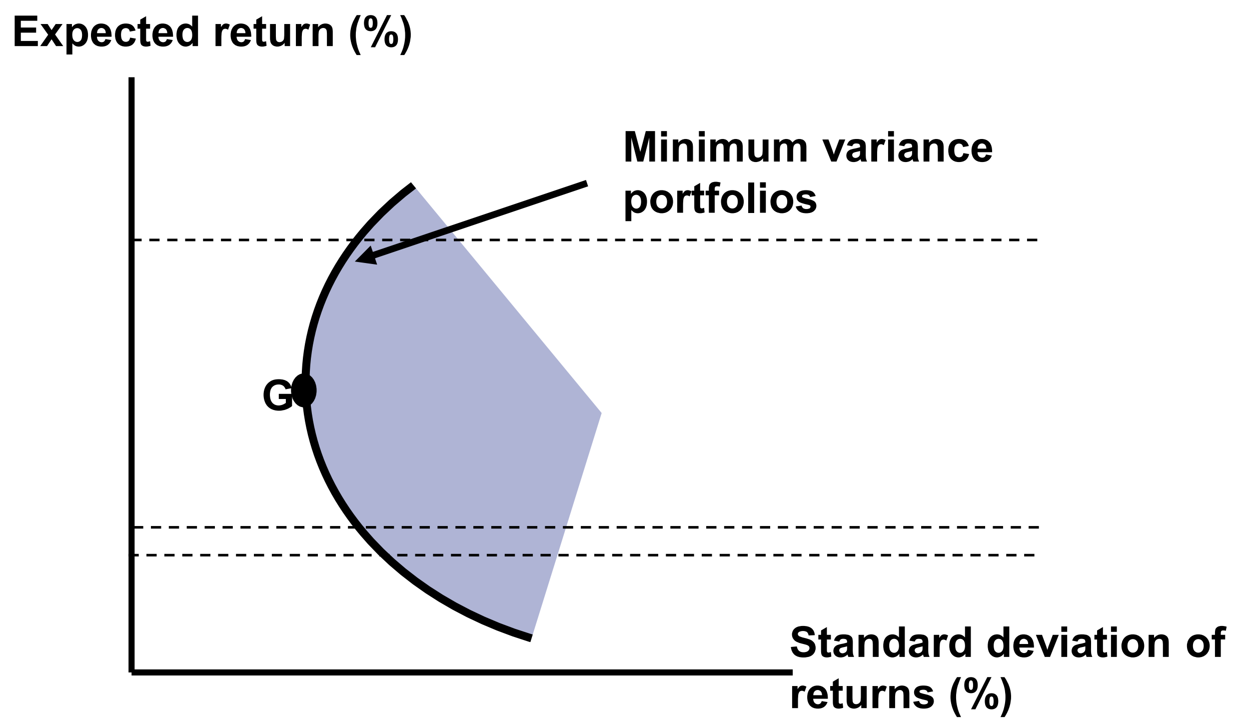
- This set of feasible portfolios is also referred to as the **opportunity set** of risky portfolios.

- Each point within the opportunity set represents a possible level of risk and return that an investor can achieve.



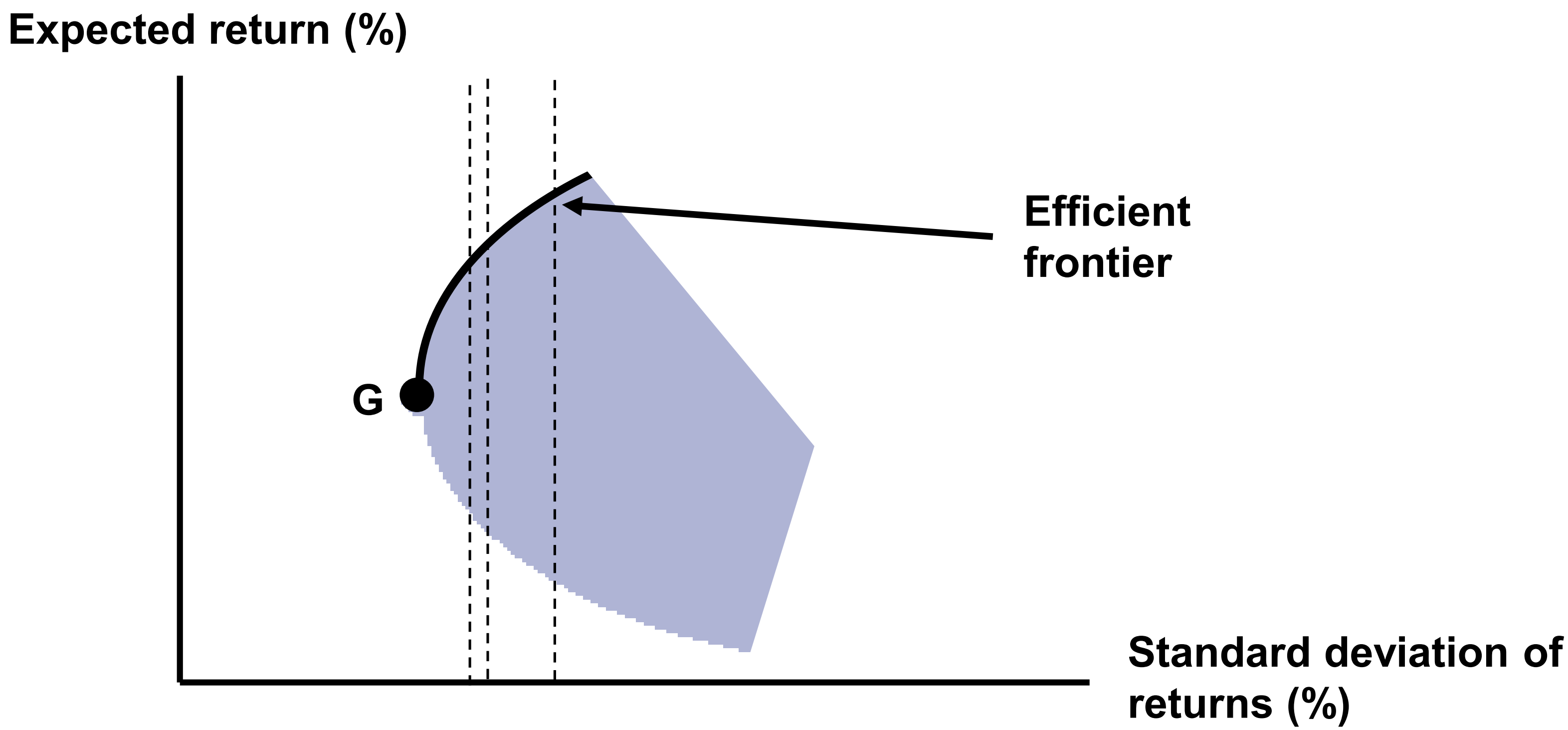
Minimum variance frontier

› The subset of feasible portfolios with the lowest risk at each level of expected return are known as minimum variance portfolios and form the **minimum variance frontier**.



Efficient frontier

› The subset of minimum variance portfolios with the highest expected returns at each level of risk are known as efficient portfolios and form the **efficient frontier**.



Efficient frontier

› Based on this analysis, a rational investor will only choose portfolios on the efficient frontier – but which portfolio?

› The portfolio chosen will depend on each individual investor's **attitude toward risk**:

- Those who are **more risk-averse** will choose portfolios close to point G – these portfolios have less risk and lower returns.
- Those who are **less risk-averse** will choose portfolios further to the right on the efficient frontier – these portfolios offer higher expected returns but carry greater risk.

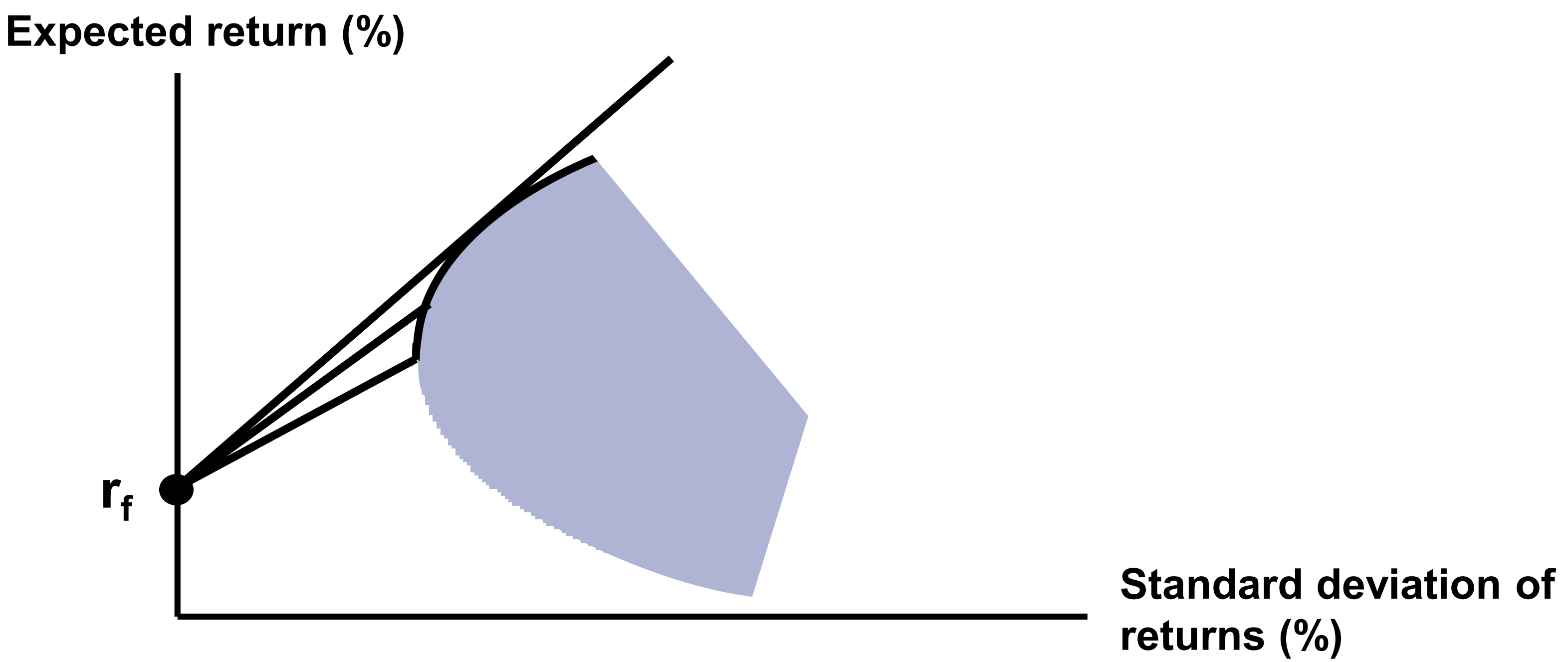
Inclusion of a risk-free asset

- The introduction of a **risk-free asset** ($\sigma = 0$) expands the risk-return opportunities available for investment. New sets of feasible portfolios now exist.

- Each of the new sets of feasible portfolios is represented by a straight line from the risk-free asset to a portfolio on the efficient frontier.

- The position of each portfolio on these lines depends on the weighting of the risk-free asset and the risky asset in the portfolio.

Inclusion of a risk-free asset



› To earn the highest possible expected return for any level of risk, find the portfolio that generates the steepest possible line when combined with the risk-free asset.

Inclusion of a risk-free asset

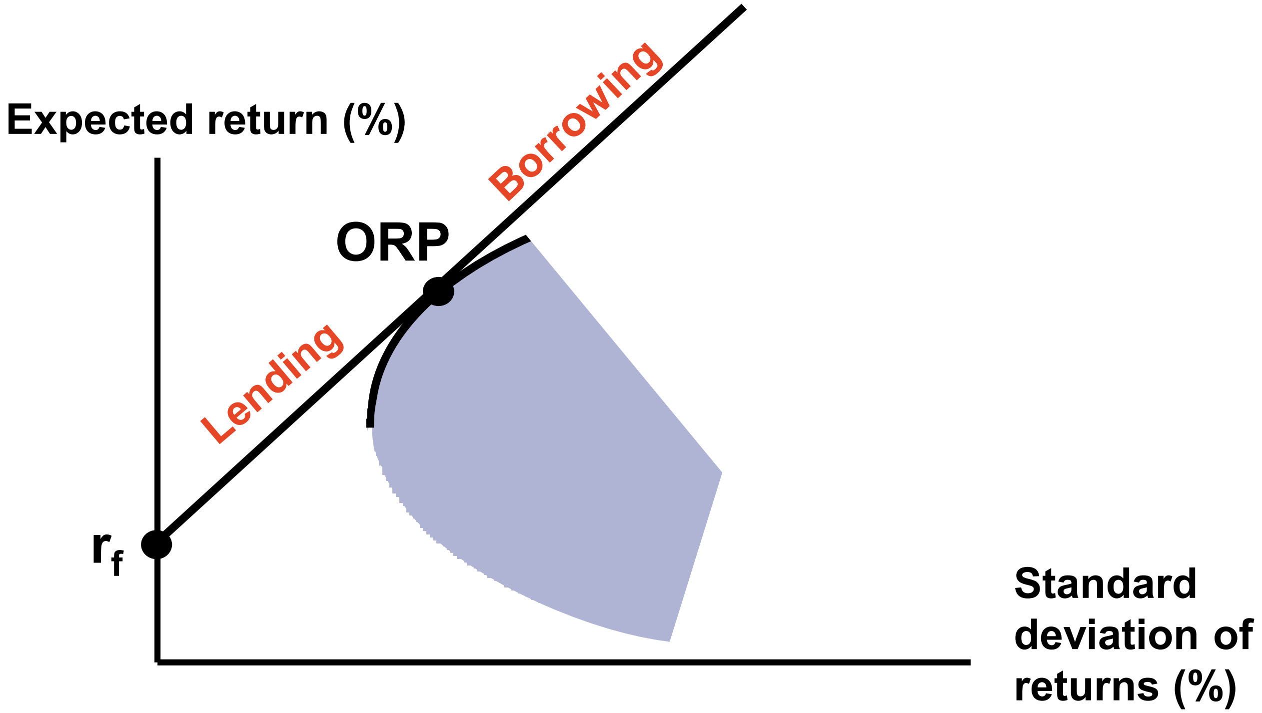
- The portfolio with the highest Sharpe ratio (reward-to-risk ratio) is the portfolio where the line with the risk-free investment is **tangential** to the efficient frontier of risky investments.

$$\text{Sharpe ratio} = \frac{r_p - r_f}{\sigma_p}$$

- The portfolio that is generated by this tangent line is known as the **optimal risky portfolio**.

Inclusion of a risk-free asset

- Inclusion of risk-free asset allows for the possibility of borrowing and lending at the risk-free interest rate (r_f).

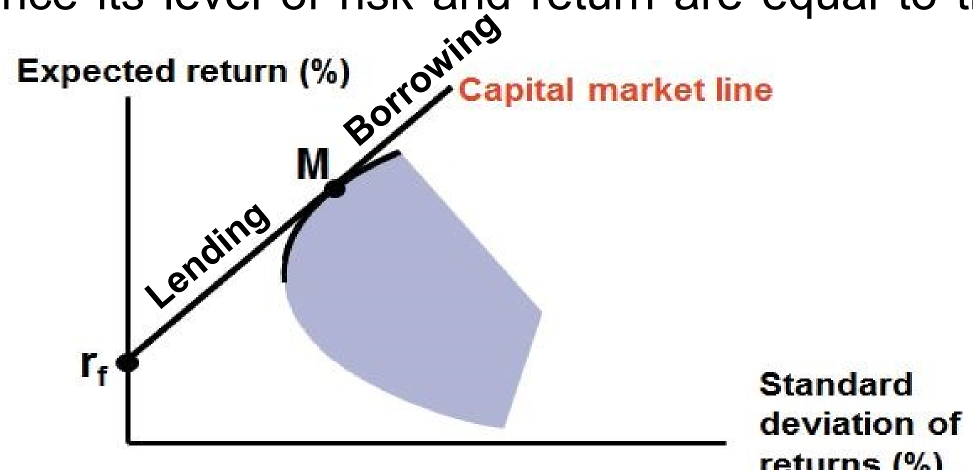


Optimal portfolio

- The optimal risky portfolio that all investors will hold is a theoretical market-valued weighted portfolio of all available risky assets.

- The optimal risky portfolio is then combined with the lending/borrowing at the r_f to obtain an exposure that best suits the investor's preferences.

- The creates a line called the **capital market line (CML)** and the point of tangency is the optimal risky portfolio and represented by the portfolio M. It is the **market portfolio**, and hence its level of risk and return are equal to those of the market overall.



The Capital Asset Pricing Model (CAPM)

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The CAPM

- The equation for the CML gives us an expression for the expected return of any efficient portfolio:

$$r_p = r_f + \frac{r_m - r_f}{\sigma_m}\sigma_p$$

where:

r_p, r_f, r_m	=	return on the portfolio, the risk-free asset and the market, respectively
σ_p, σ_m	=	standard deviation of the portfolio and the market, respectively

- This equation can be rearranged to give:

$$r_p = r_f + \left(r_m - r_f\right)\frac{\sigma_p}{\sigma_m}$$

The CAPM

- The expected ratio of the standard deviation of the portfolio to that of the market (σ_p/σ_m) is the beta (β) of the *portfolio*:

$$E(r_p) = r_f + E(r_m - r_f)\beta_p$$

- To describe how *individual securities* are priced:

$$E(r_i) = r_f + E(r_m - r_f)\beta_i$$

where:

r_f	= the risk-free rate of return
$E(r_p)$, $E(r_i)$	= expected return on the portfolio and security *i*, respectively
$E(r_m)$	= expected return on the market
β_p, β_i	= beta of the portfolio and security *i*, respectively

The CAPM

- This equation can be used to determine the expected return of any security.

- It is consistent with the notion that the expected return, and hence the price, of a security is determined by its market risk.

- This is because rational investors will hold a well-diversified portfolio and the only risk of a well-diversified portfolio is market risk.
  - Unique risk is diversifiable and does not warrant extra returns.

The CAPM

› The CAPM states that the expected return on any investment is equal to the risk-free rate of return plus a risk premium proportional to the amount of market risk in the investment.

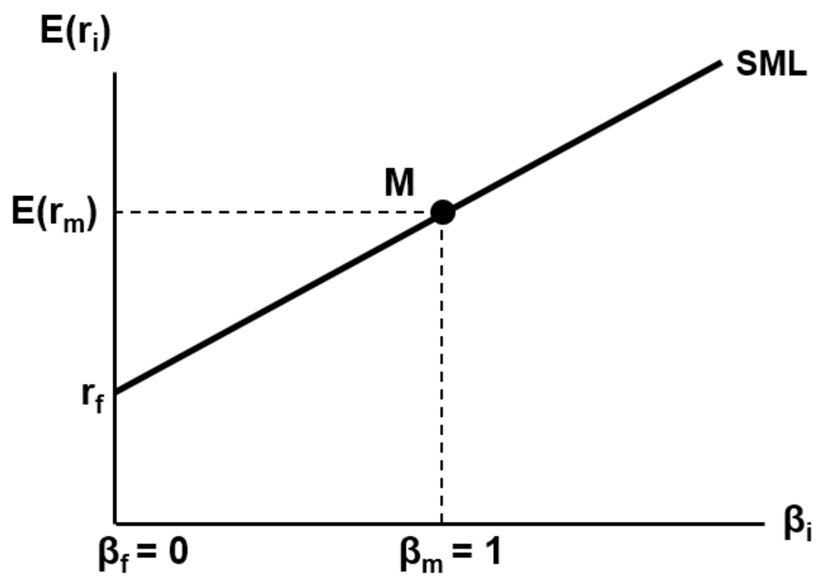
$$E(r_i) = r_f + \text{ risk premium}$$

$$\begin{aligned} &\text{risk premium} \\ &= \text{quantity of risk} \quad \times \text{ price of risk} \\ &= \beta_{\mathrm{p}} \times E\left(r_m - r_f\right) \end{aligned}$$

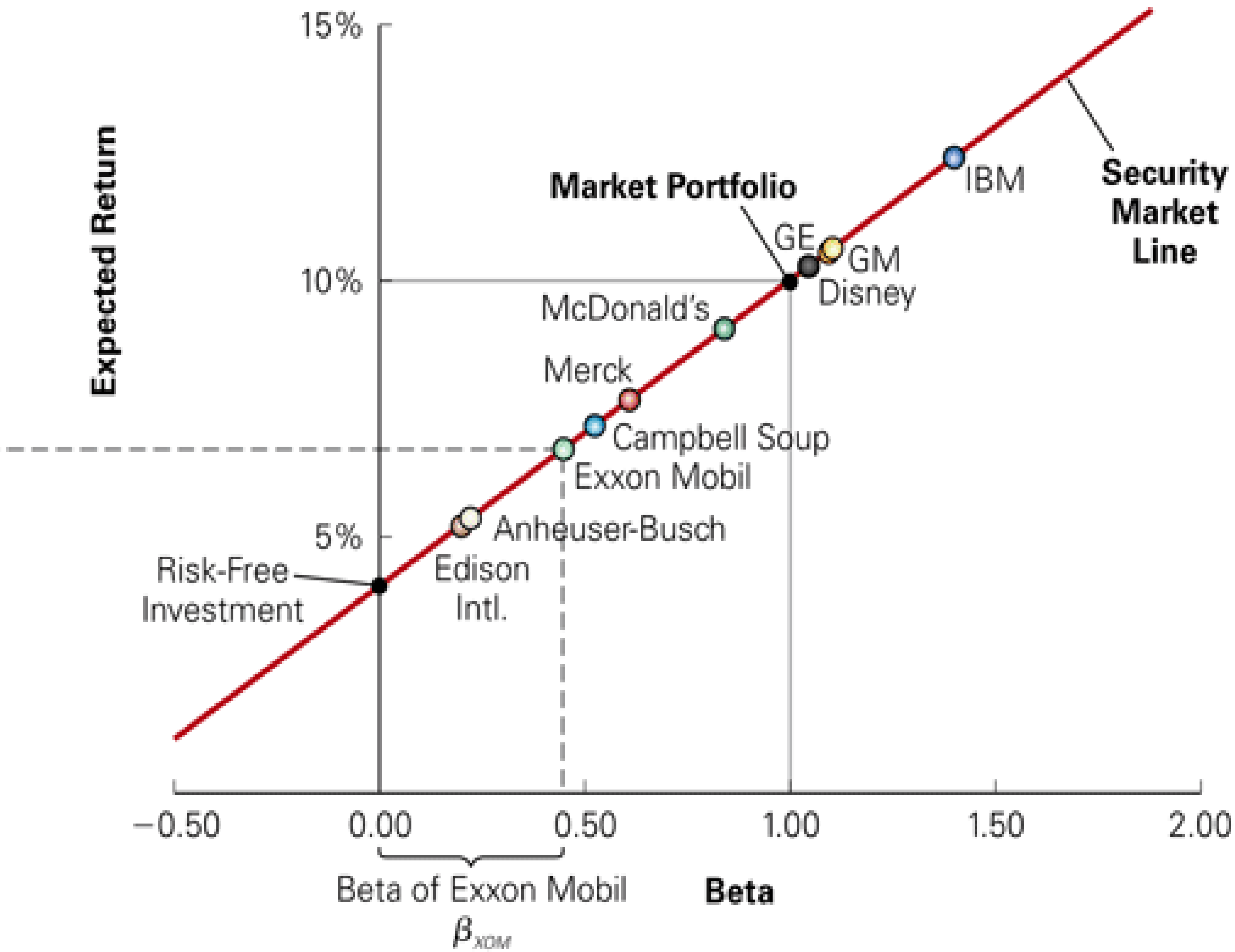
› The quantity of market risk attributable to an asset is given by its β. For each unit of market risk faced, the investor expects to be remunerated by a market rate of return over and above the risk-free rate ($r_m - r_f$).

The security market line

- The CAPM can be graphically represented by a relationship known as the **security market line**.
- Once the risk-free rate and the expected return on the market have been estimated, the expected return on a security can be calculated as a function of its beta and should plot on the SML.



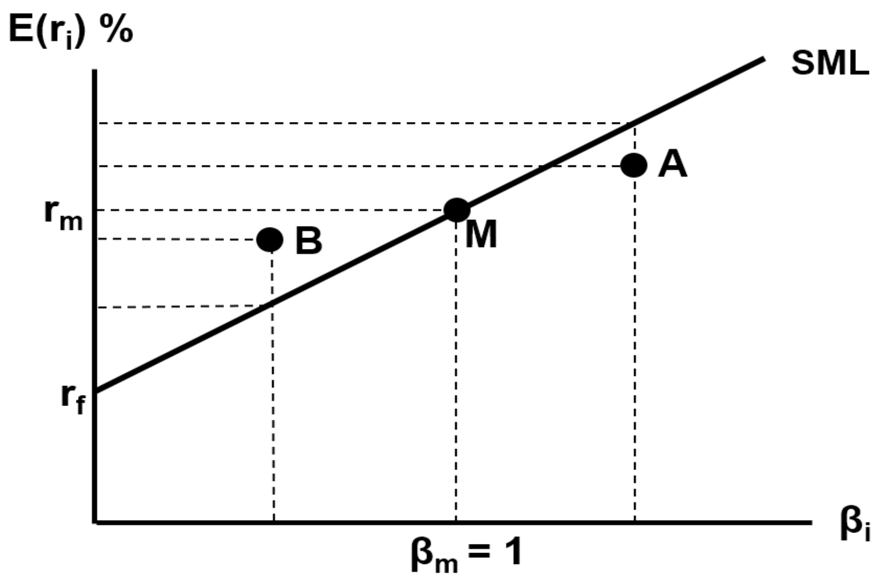
The security market line



The security market line

- In equilibrium, no securities can lie below or above the CAPM line.

- Neither A nor B are priced at equilibrium – this disequilibrium will be only temporary.



Estimation of beta

- In order to determine the expected return on a security, estimates are required of:
  - The risk-free rate (r_f)
  - The expected return on the market in excess of the risk-free rate ($r_m - r_f$)
  - The securities' beta (β_i)

Estimation of beta

- Since β is the relationship between returns on a security and the market, it can be estimated by quantifying this historical relationship.

- The equation for the CAPM can be more generally as:

$$\boxed{Y_t = a + bX_t}$$

where:

Y_t = $E(r_{i,t} - r_{f,t})$

X_t = $E(r_{m,t} - r_{f,t})$

$r_{i,t}$ = the return on security *i* earned over period *t*

$r_{f,t}$ = the risk-free rate of return earned over period *t*

$r_{m,t}$ = the market rate of return earned over period *t*

Estimation of beta

› The CAPM implies that the coefficient *a* = 0.

› The coefficient *b*, which measures the *β* of security *i*, can be calculated from historical data, and is given by:

$$\beta = \frac{\text{covariance}\left(X_t, Y_t\right)}{\text{variance}\left(X_t\right)}$$

$$\beta_{i,m} = \frac{\sigma_{i,m}}{\sigma_m^2}$$

› The first step in calculating beta is to collect historical data and calculate a sequence of **excess returns on a security** and **on the market**.

Estimation of beta

› The **returns on a security** (must include the dividend paid, if any) is:

$$r_{i,t} = \frac{P_t - P_{t-1} + Div_t}{P_{t-1}}$$

› The **returns on the market** is:

$$r_{m,t} = \frac{I_t - I_{t-1}}{I_{t-1}}$$

Estimation of beta

› Once the excess returns on the security and market have been calculated by subtracting the risk-free rate from the security and market returns, the **covariance of the excess returns on the security and market** can be calculated using the following expression:

$$\sigma_{i,m} = \frac{\sum\left[\left\{\left(r_{i,t} - r_{f,t}\right) - \left(\overline{r_{i,t} - r_{f,t}}\right)\right\} \times \left\{\left(r_{m,t} - r_{f,t}\right) - \left(\overline{r_{m,t} - r_{f,t}}\right)\right\}\right]}{n-1}$$

Estimation of beta

› The **variance of excess returns on the market** is:

$$\sigma_m^2 = \frac{\sum\left[\left\{\left(r_{m,t} - r_{f,t}\right) - \left(\overline{r_{m,t} - r_{f,t}}\right)\right\}\right]^2}{n-1}$$

› The **beta** is then the ratio of the covariance between excess security returns and excess market returns to the variance of excess market returns.

$$\beta_{i,m} = \frac{\sigma_{i,m}}{\sigma_m^2}$$

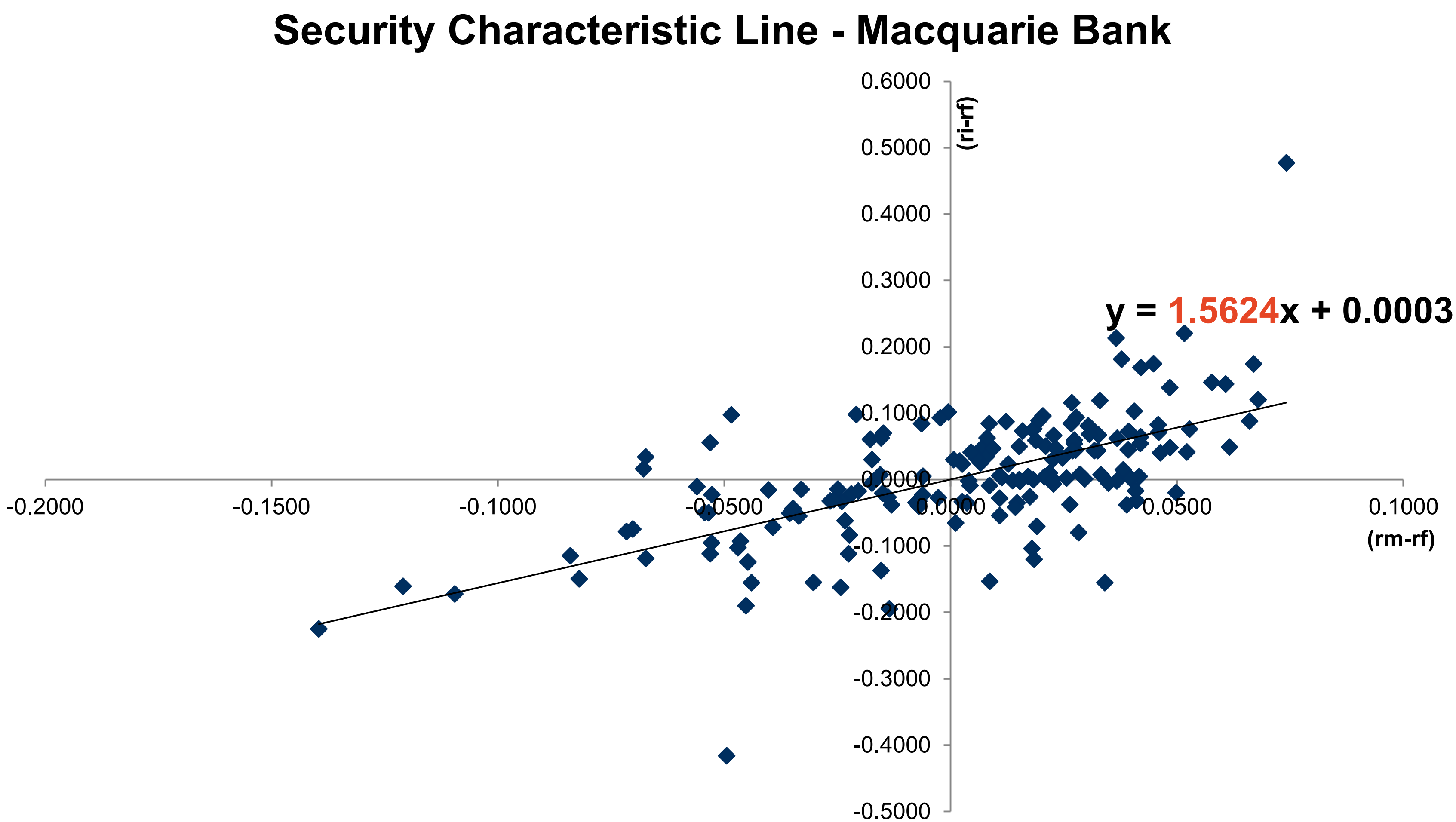
Estimation of beta

- The beta of a security can also be estimated using a simple linear regression.

- This is further illustrated by considering the **security characteristic line**, which is the line that best fits the relationship between excess returns on the security and the market.

- The slope of the characteristic line is the **beta** of the security.

Estimation of beta



Adjusting beta for leverage

- Often estimation of beta is problematic because the company:
  - has insufficient price history
  - has undergone a structural change
  - is considering investment in a new business

- In such cases, there is a need to estimate β from another company and adjust its β to reflect different capital structure.

Adjusting beta for leverage

› Since the debt of a company increases the cash flow risk of shareholders, it is necessary to separate the financial risk of a levered company from its systematic risk:

$$\beta_l = \beta_u[1 + (1 - t_c)\phi]$$

where:

β_l	=	levered beta of a corporation
β_u	=	unlevered beta of a corporation
t_c	=	corporation tax rate
Φ	=	debt/equity ratio

Does the CAPM work?

- Common methodology of empirical tests:
  - Time-series regression of individual stocks against (a proxy for) the market portfolio over a specific period
  - Divide all stocks into *n* portfolios based on β
  - At the end of each year, β's are re-estimated and portfolios re-balanced

- An early paper by **Fama and MacBeth (1973)** using stocks from the New York Stock Exchange (NYSE) over a 1926-1968 demonstrated the existence of a positive relationship between beta and average stock returns.

Does the CAPM work?

› **Fama and French (1992)** carried out a series of tests using monthly prices from the NYSE, the American Stock Exchange and the NASDAQ.

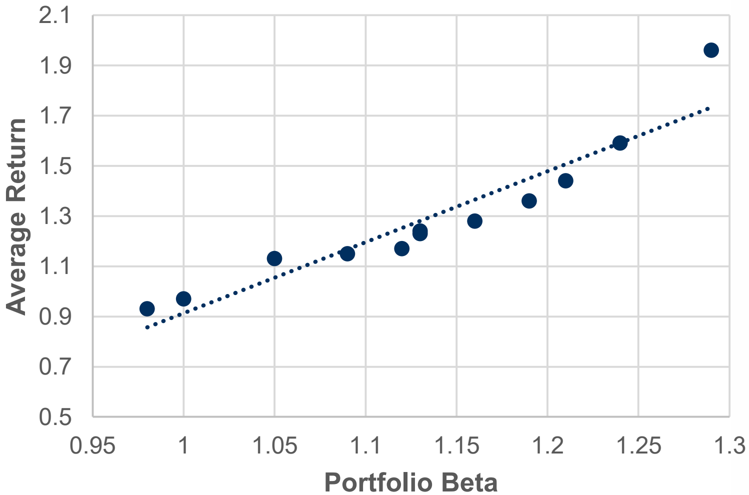


Table AI

Average Returns, Post-Ranking βs and Fama-MacBeth Regression Slopes for Size Portfolios of NYSE Stocks: 1941–1990

At the end of each year $t-1$, stocks are assigned to 12 portfolios using ranked values of ME. Included are all NYSE stocks that have a CRSP price and shares for December of year $t-1$ and returns for at least 24 of the 60 months ending in December of year $t-1$ (for pre-ranking β estimates). The middle 8 portfolios cover size deciles 2 to 9. The 4 extreme portfolios (1A, 1B, 10A, and 10B) split the smallest and largest deciles in half. We compute equal-weighted returns on the portfolios for the 12 months of year t using all surviving stocks. Average Return is the time-series average of the monthly portfolio returns for 1941–1990, in percent. Average firms is the average number of stocks in the portfolios each month. The simple βs are estimated by regressing the 1941–1990 sample of post-ranking monthly returns for a size portfolio on the current month's value-weighted NYSE portfolio return. The sum βs are the sum of the slopes from a regression of the post-ranking monthly returns on the current and prior month's VW NYSE returns.

The independent variables in the Fama-MacBeth regressions are defined for each firm at the end of December of each year $t-1$. Stocks are assigned the post-ranking (sum) β of the size portfolio they are in at the end of year $t-1$. ME is price times shares outstanding at the end of year $t-1$. In the individual-stock regressions, these values of the explanatory variables are matched with CRSP returns for each of the 12 months of year t. The portfolio regressions match the equal-weighted portfolio returns with the equal-weighted averages of β and ln(ME) for the surviving stocks in each month of year t. Slope is the average of the (600) monthly FM regression slopes and SE is the standard error of the average slope. The residuals from the monthly regressions for year t are grouped into 12 portfolios on the basis of size (ME) or pre-ranking β (estimated with 24 to 60 months of data, as available) at the end of year $t-1$. The average residuals are the time-series averages of the monthly equal-weighted portfolio residuals, in percent. The average residuals for regressions (1) and (2) (not shown) are quite similar to those for regressions (4) and (5) (shown).

	Portfolios Formed on Size											
	1A	1B	2	3	4	5	6	7	8	9	10A	10B
Ave. return	1.96	1.59	1.44	1.36	1.28	1.24	1.23	1.17	1.15	1.13	0.97	0.93
Ave. firms	57	56	110	107	107	108	111	113	115	118	59	59
Simple β	1.29	1.24	1.21	1.19	1.16	1.13	1.13	1.12	1.09	1.05	1.00	0.98
Standard error	0.07	0.05	0.04	0.03	0.02	0.02	0.02	0.02	0.01	0.01	0.01	0.01

Does the CAPM work?

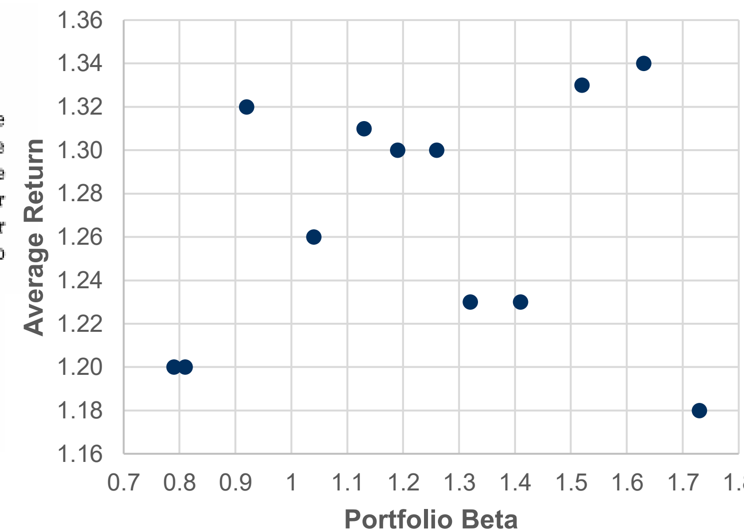
› But the relationship between β and average return disappears during the more recent 1963–1990 period.

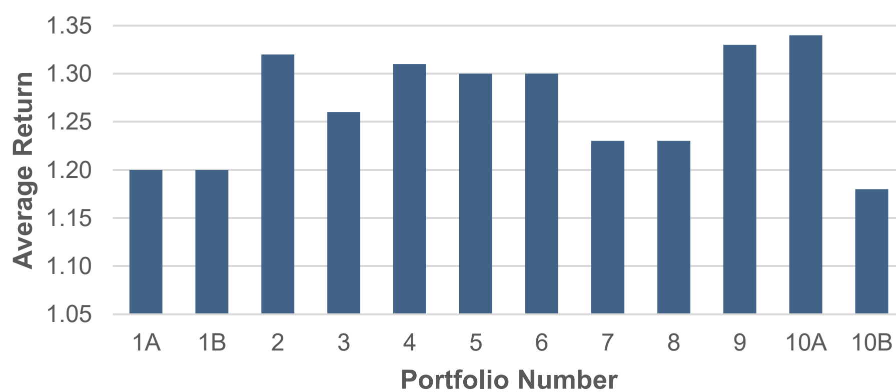
Table II

Properties of Portfolios Formed on Size or Pre-Ranking β: July 1963 to December 1990

At the end of June of each year t, 12 portfolios are formed on the basis of ranked values of size (ME) or pre-ranking β. The pre-ranking βs use 2 to 5 years (as available) of monthly returns ending in June of t. Portfolios 2–9 cover deciles of the ranking variables. The bottom and top 2 portfolios (1A, 1B, 10A, and 10B) split the bottom and top deciles in half. The breakpoints for the ME portfolios are based on ranked values of ME for all NYSE stocks on CRSP. NYSE breakpoints for pre-ranking βs are also used to form the β portfolios. NYSE, AMEX, and NASDAQ stocks are then allocated to the size or β portfolios using the NYSE breakpoints. We calculate each portfolio's monthly equal-weighted return for July of year t to June of year $t+1$, and then reform the portfolios in June of $t+1$.

	1A	1B	2	3	4	5	6	7	8	9	10A	10B
Panel B: Portfolios Formed on Pre-Ranking β												
Return	1.20	1.20	1.32	1.26	1.31	1.30	1.30	1.23	1.23	1.33	1.34	1.18
β	0.81	0.79	0.92	1.04	1.13	1.19	1.26	1.32	1.41	1.52	1.63	1.73





Does the CAPM work?

- One of the implications of the CAPM is that the return on stocks with smaller betas should be lower than returns on stocks with large betas – Fama and French found **no statistically significant relationship between average returns and** β.

- Another implication of the CAPM is that there should be no relationship between any other (risk) factor(s) and average returns.

Does the CAPM work?

› **Fama and French (1992)** found that returns on the largest stocks are smaller than the returns on the smallest stocks.

	1A	1B	2	3	4	5	6	7	8	9	10A	10B
	Panel A: Portfolios Formed on Size											
Return	1.64	1.16	1.29	1.24	1.25	1.29	1.17	1.07	1.10	0.95	0.88	0.90

› **Fama and French (1993)** propose that two other factors also have an influence on stock returns – a three-factor model.

$$r_i = r_f + \beta_1(r_m - r_f) + \beta_2(r_s - r_b) + \beta_3(r_h - r_l)$$

Does the CAPM work?

- Many studies have extended the original Sharpe-Lintner CAPM giving rise to extensive literature in asset pricing…
  - Generic risk factors (arbitrage pricing theory): **Ross (1976)**
  - No risk-free asset (zero-beta CAPM): **Black (1972)**
  - Multi-period consumption and changes in future returns (intertemporal CAPM): **Merton (1973)**
  - Non-traded assets: **Mayers (1972)**
  - International investing and foreign currency exposures: **Solnik (1976)**

- … and multi-factor investing: **Harvey, Liu and Zhu (2016)**
  - Hundreds of factors have been proposed!
  - Which one(s) can completely explain cross-sectional variation in expected returns???

Conclusion

› Possible portfolio risk and return outcomes.

- Feasible portfolios
- Minimum-variance and efficient frontier
- Optimal risky portfolio
- Inclusion of risk-free asset

› The Capital Asset Pricing Model.

- Derivation of the equation
- Estimation of the parameters of the CAPM
- Evaluation of the CAPM: empirical evidence

› **Next lecture: Company Cost of Capital**

The End

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