**3. Methodology**

**3.1 Arithmetic expression of PPP**

The PPP relationship is

Where the variables are

P = domestic currency (Japanese yen) price level

P\* = foreign currency (American dollar) price level

E = the exchange rate, the domestic currency (Japanese yen) price of a unit of foreign (American dollar) currency

K = a constant

This definition of PPP is Relative PPP, which is a lighter version of Absolute PPP. Since we bring in a constant value K, it makes allowance for the difference between P and EP\*.

**3.2 Time series linear regression model**

Time series regression models are popular models that attempt to explain the present response by using the response history and the transfer of dynamics from relevant predictors (Greene, 2008). There are different representations of the models since the relationships among variables can be diverse, forming different theoretical frameworks.

One of the most popular models is time series linear regression model. By analysing time series data, we want to examine the linear relationship among variables. In our case, we want to examine the linear relationship between the domestic price level and the foreign price level when transferred to the domestic currency.

**3.2.1 Unit root test**

Usually, ordinary least squares (OLS) is used to estimate coefficient of slope in autoregressive models. However, the use of OLS depends on that stochastic processes are stationary. Ineffective estimate would happen in OLS if the stochastic processes are non-stationary. Such estimates are called ‘spurious regression’ results by Granger and Newbold (1974). They argued there is no economic meaning even if the R2 value and t-ratio are both high.

To estimate coefficient of slope, the first step is to operate unit root test where the null hypothesis is that unit root exists. If the null hypothesis is rejected, it means there is no unit root and we can continue to use OLS. However, if the presence of unit root is not rejected, then we need to use the difference operator.

In probability theory and statistics, the unit root is a feature of some stochastic processes (such as random walks) that tend to produce some problems during the statistical inference with respected to the time series models. A linear stochastic process has a unit root if 1 is the root of the process’s characteristic equation. This process is not stationary but does not always have a trend.

If other roots of the characteristic equation locate in the unit circle—which means modulus (absolute value) is less than 1, then the first difference of the process is stationary; otherwise, multiple differences of the process are needed to stay stationary. Due to the feature, unit root process is also introduced as difference stationary.

The presence of a unit root can tested using a unit root test.

Consider a discrete-time stochastic process , and suppose that it can be written as an autoregressive process of order p:

Here, is a serially uncorrelated, zero-mean stochastic process with constant variance . For convenience, assume . If m = 1 is a root of the characteristic equation:

Then the stochastic process has a unit root or, alternatively, is integrated of order one, denote I(1). If m = 1 is a root of multiplicity r, then the stochastic process is integrated of order r, denoted I(r).

There are variety of unit root test, and in our case, we use the ADF test. If the variables of domestic price level and the foreign price level when transferred to the domestic currency are stationary, then we can use OLS; otherwise, we should employ the further test which called co-integration test.

**3.2.2 Co-integration test**

Co-integration test is applied when analysing non-stationary time series. Tests for co-integration tend to identify if there exists stable, long-run relationship between sets of variables. To understand co-integration, it is necessary to identify the definition of integration.

A series with no deterministic component which has a stationary, invertible, ARMA representation after differencing d times, is said to be integrated of order d, denoted .

After knowing this basic concept, then we can more easily understand that the components of the vector are said to be co-integrated of order d, b, denoted , if (i) all components of are ; (ii) there exists a vector so that . The vector is called the co-integrating vector (Engle and Granger, 1987).

For the study of PPP theory, the vector time series has two elements and can be written as

 (5)

Previous studies have found that the logarithms of , and are integrated of order 1. The logarithms of and are in fact series with single unit roots over the sample in this study.

The normalized vector is defined as

(6)

Co-integration requires that when multiplied by vector time series mentioned above

Where is stationary (integrated of order ). A stationary variable has finite variance, a constant mean, and a tendency to return to that mean in a finite length of time. If is stationary, then PPP is a meaningful concept. The difference between and its mean is interpreted as the percentage deviation of the domestic price level or the exchange rate from the PPP equilibrium level. The null hypothesis is that the bivariate time series , is not co-integrated. If this null hypothesis is rejected, then the PPP relationship receives empirical support.

**3.2.3 Error correction model**

Engle and Granger (1987) proved that if is co-integrated, then the change in has an error-correction representation. Davidson et al. (1978) also proposed that error correction model allows for much volatility in short-term dynamics while the model is forced to return to long-term equilibrium.

The general error correction model can be expressed as

 (7)

Where B is the backshift operator. The following results were included in the Granger’s Representation Theorem (1987):

1. .

2. is a stationary stochastic process.

3. In there exists some lag length where is unity.

4. is multivariate white noise.

5. is defined as deviations from the co­-integrating regression.

Expression (7) is the error correction mechanism for any co-integrated vector time series . The particular set of error correction models estimated and their parameter values depend on whether the nominal exchange rate regime is fixed and flexible. In our case, the exchange rate regime is flexible between Japanese yen and US dollar.

Actually, the domestic price level can change in a flexible exchange rate regime. The error correction mechanism for the change in the domestic price level, the inflation rate, using (7) and (5) is

 (8)

Where and are adequate to reduce the appropriate element of the error term to white noise. the term in the representation is the lagged residual from the co-integrating regression.

 (9)

The lagged residual from this regression, , is the error correction term, which is the estimated lagged deviation from PPP. If is positive and significantly different from zero, then the domestic price level tends to return to the long-term PPP level. From Stock (1987), (9) produces consistent estimate of and . Engle and Granger (1987) proposed the two-step method to estimate the (8) and (9) is consistent and asymptotically efficient.

The domestic price level is believed to be adjusted to the level under PPP relationship no matter the exchange rate regime is fixed or flexible. However, it is expected that the more the authorities allow the nominal exchange rate to change, the less the error correction are required to hold the PPP relationship. In our case, the exchange rate between US dollar and Japanese yen is flexible, so we have to modify our model.

Then the cointegrating regression should be written as

 (10)

The error correction model of the change in the exchange rate is

 (11)

Similar with (8), the term is the error correction term to force the nominal exchange rate to move towards to the value corresponding to PPP relationship. Equation (11) can be estimated by using the time period data when the exchange rate is allowed to change, and this is consistent to our case.

If PPP holds and , the error correction terms in (8) and (11) are different normalizations on the same equilibrium relationship. In practice, the residuals from regression (9) and (10) are almost equal in magnitude and opposite in sign (Johnson, 1990). Our error correction model (11) is estimated with the residuals from (9). This allows the co-integrating regression to be estimated from the longest possible time series, as well as allows the error correction model to adjust to different exchange rate regime (Hendry, 1986). Both of the error correction model of prices (8) and exchange rate (11) use the residuals from (9) as the error correction terms. In other words, it means the coefficient on the error correction term , should have opposite sign in equation (8) and (11). If the residual in (9) is positive, Japanese goods are expensive relative to American goods, and in other words, Japanese price tend to fall in (8) or the exchange rate (Japanese yen per U.S. dollar) in (11) tends to depreciate.

**3.3 Source of data**

Our case needs three sets of data: exchange rate (Japanese yen per U.S. dollar), national price level of Japan and the national price level of the U.S.. The exchange rate is easily accessible in IMF Monthly International Financial Survey. The real confusion is the choice of data of national price level. Some of the literatures used consumer prices as the national price level. Pelagatti and Colombo (2015) employed the CPI index since they argued it is an aggregate data that can be calculated to real exchange rate. Chen and Wu (2000) focused on the Consumer Price Index as well since the data can directly reflect the price level in a specific country. However, from ERDEY and FÖLDVÁRI (2009), producer prices are more adequate than consumer prices to test the PPP theory. As mentioned before, the non-traded goods may cause the deviation from PPP theory, so we prefer the indices that mainly include the traded goods. Since in producer price indices the traded goods have a much higher weight, especially in countries that have a high export openness such as the U.S. and Japan, Producer Price Indices (PPI) could reflect the price of traded goods more accurately. Therefore, we determine to use the PPI in the U.S. and Japan, which can also be obtained in IMF Monthly international Financial Survey. Meanwhile, some scholars employ other indices as price level, for example, Edison (1987) considered the consistently calculated and continuously available, and chose GDP price deflators as national price level. In our case, we do not consider this kind of index. After determining the category of the data in our case, then we should choose the specific range and frequency of the data. Since the daily data is relatively stable or sometimes no changes and the yearly data could not reflect the change, we finally choose the monthly data and find it suitable. On the other hand, we choose 10 years data and there are 120 data for each variable. It is a suitable sample and we obtained the monthly (January 2009-December 2018) series of JPY/USD spot exchange rate and PPI in Japan and the U.S. respectively.

**4. Empirical results**

**4.1 Regression analysis**

First of all, the hypothesis of our case is that we believe in the existence of deviation from PPP in the short run. The empirical experiment in our study is to assess if the PPP holds in the long run or if there exists a equivalent relationship between exchange rate and purchasing power.

Before doing the regression model, we could state some descriptive results. From the figure 1, where the monthly exchange rate of change of the JPY/USD exchange rate is plotted against the difference of the monthly growth rates of the PPI in Japan and the U.S. in the January 2009-December 2018, we can roughly observe a relatively positive relationship.

*Figure1*



 *Figure2 spot exchange rate of JPY/USD* *figure 3 PPI (blue: Japan; red, the U.S.)*

Figure 2 and figure 3 represent the spot exchange rate of JPY/USD and the PPI in the two countries respectively. The exchange rate have a declined trend before 2012 and begin to rise afterwards. The PPI in Japan have a relatively little volatility with a stable value, while the PPI in the U.S. have a strong rising trend. On the other hand, if we look at the two figures together, we can find when the exchange rate is declining, the PPI in Japan is higher than that in the U.S. and vice versa.

Then we set up the simple regression model

Where is denoted as the exchange rate (JPY/USD), is denoted as the PPI in Japan and is denoted as the PPI in the U.S.. The results is showed as below.

 (9.5188) (-5.6657)

The regression has an opposite result compared to the result of ERDEY and FÖLDVÁRI (2009), who use the same simple regression model. The coefficient of proportioning PPI has an opposite sign to that in ERDEY and FÖLDVÁRI (2009). However, this regression result makes little sense since the PPP does not hold in the short run and the PPI is not the value of purchasing power. We do the regression model because we would do the relative test to assess the long-run equivalent relationship.



**4.2 Unit root test**

Before doing the following tests, we should process the data first. The same as the most previous literature, we used the logarithm of the variables. The advantage of the this process is that

There are various unit root tests, and we choose an augmented Dickey-Fuller test. The null hypothesis of the ADF test is that there exists a unit root—which means the time series variable is not stationary. The method here is not similar with that in the study of ERDEY and FÖLDVÁRI (2009). The method here is stricter. The criteria is comparing the value of Akaike info criterion (AIC), Schwarz criterion (SC) and Hannan-Quinn criterion (HQC) under three conditions which is trend and intercept, intercept and none. We should focus on the condition that has the most of smallest value among them.

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  |
| LnS | trend&intercept | intercept | none |
| AIC | -4.815442 | -4.808403 | -4.816234 |
| SC | -4.72152 | -4.737962 | -4.769273 |
| HQC | -4.777307 | -4.779802 | -4.797167 |
| p-value | 0.6407 | 0.7483 | 0.8265 |

*Table 1*

|  |  |  |  |
| --- | --- | --- | --- |
| LnPPIJ | trend&intercept | intercept | none |
|  |  |  |  |
| AIC | -8.026022 | -8.031564 | -8.025199 |
| SC | -7.9321 | -7.961123 | -7.978238 |
| HQC | -7.987887 | -8.002963 | -8.006131 |
| p-value | 0.6302 | 0.4562 | 0.7845 |

*Table 2*

|  |  |  |  |
| --- | --- | --- | --- |
| LnPPIU | trend&intercept | intercept | none |
| AIC | -8.381019 | -8.366299 | -8.314114 |
| SC | -8.287098 | -8.295858 | -8.267153 |
| HQC | -8.342884 | -8.337698 | -8.295046 |
| p-value | 0.1891 | 0.0588 | 1 |

*Table 3*

From table 1, we find that all of the AIC, SC and HQC has the smallest value under the condition of none of trend and intercept. So we choose to observe this situation and the result shows that the null hypothesis is not rejected. This means that the time series of LnS has a unit root under none of trend and intercept. The similar analysis can be applied to the variable of LnPPIJ and LnPPIU. LnPPIJ has a unit root under none of trend and intercept and LnPPIU has a unit root under the trend and intercept. These results showed that all of the three variables are non-stationary.

|  |  |  |  |
| --- | --- | --- | --- |
| ΔLnS | trend&intercept | intercept | none |
| AIC | -4.800744 | -4.816527 | -4.830863 |
| SC | -4.730303 | -4.769566 | -4.807383 |
| HQC | -4.772143 | -4.797459 | -4.821329 |
| p-value | 0 | 0 | 0 |

*Table 4*

|  |  |  |  |
| --- | --- | --- | --- |
| ΔLnPPIJ | trend&intercept | intercept | none |
| AIC | -8.010672 | -8.025238 | -8.04109 |
| SC | -7.940231 | -7.978277 | -8.01761 |
| HQC | -7.982071 | -8.00617 | -8.031557 |
| p-value | 0 | 0 | 0 |

*Table 5*

|  |  |  |  |
| --- | --- | --- | --- |
| ΔLnPPIU | trend&intercept | intercept | none |
| AIC | -8.32999 | -8.31647 | -8.170956 |
| SC | -8.259549 | -8.269509 | -8.147475 |
| HQC | -8.301389 | -8.297403 | -8.161422 |
| p-value | 0 | 0 | 0 |

*Table 6*

Then we should operate the first difference of these variables, from the results, we find the null hypothesis is rejected in all three time series data after processing. So we get the conclusion that these three variables are , which means under the first difference, these variables are stationary.

In this situation, when we use the data set to operate the regression, we tend to get a ‘spurious regression’, then the co-integration test is essential.

**4.3 Co-integration test**

Since the regression is a ‘spurious regression’, we can not conclude from the regression result. Next we would operate the co-integration test to assess if there exists a long run equivalent relationship or purchasing power parity.

As a first step, we carry out the Engle-Granger test on PPP theory.

Ln St = α0 + α1 lnPPItJ + α2 lnPPItU +ut

Where we assume the error-terms, u, are assumed to be stationary.

After the regression process, we can get the residuals series, then we test the unit root of residuals under none of trend and intercept. The result is showed below, and the null hypothesis is rejected with the confidence level of 5%. It confirms that the residuals series are stationary and there exists co-integration relationships among the variables.

*Table 7. ADF test in residual*

|  |  |  |
| --- | --- | --- |
| Augmented Dickey-Fuller test statistic | -2.315446 | 0.0205 |
| Test critical values: | 1% level |  | -2.584877 |  |
|  | 5% level |  | -1.943587 |  |
|  | 10% level |  | -1.614912 |  |

We can get a preliminary statement that the purchasing power parity holds in the long term. Actually, the co-integration results just has a statistic meaning with no economic meaning, so we should be cautious with the results.

**4.4 Error correction model**

The co-integration results just show the long-run equivalent relationship between exchange rate of JPY/USD and the purchasing power and do not explain the short-run situation. So we need to process the error correction to see if the equivalent relationship exists in the short run.

Since the variables are all , then we choose the first difference of the variables to operate the error correction model.

From the figure, we can see that up to three of lag length is applied to the model, however, the results are not significant. All the first difference of variables do not pass the significance test. On the other hand, the squared R is relative too small, which means the fitting degree is not very well. From the result, we can not get the conclusion that there exists a equivalent relationship in PPP after the error correction.

|  |  |  |
| --- | --- | --- |
|  | ∆S |  |
| Constant | 0.001009 | （-0.394058） |
| ∆(PPIJ(-1)) | -0.419468 | （-0.83313） |
| ∆(PPIJ(-2)) | -0.529638 | （-1.0191） |
| ∆(PPIJ(-3)) | 0.510927 | （1.039137） |
| ∆(PPIU(-1)) | -0.302496 | （-0.543124） |
| ∆(PPIU(-2)) | 0.318053 | （0.570332） |
| ∆(PPIU(-3)) | -0.265655 | (-0.487882) |
| ∆(S(-1)) | 0.275859 | (-2.741169) |
| ∆(S(-2)) | 0.169652 | (-1.686628) |
| ∆(S(-3)) | -0.036118 | (-0.366416) |
| ECM(-1) | -0.039488 | (-2.255576) |
|  |  |  |
| R2 | 0.172237 |  |
| DW | 1.997813 |  |
| SER | 0.020984 |  |

*Table 8 error correction results*

**4.5 Discussion**

There are two question to be answered in our case. The first is if the purchasing power parity holds in the long term. The second question is if there exists an equivalent relationship between exchange rate and purchasing power in the short run.

For the first question, in our empirical evidence, the co-integration test confirms a statistic equivalent relationship in the PPP in a long term. However, the result is not reliable since the result has no economic meaning or reality meaning. For this reason, we use the error correction model to test the situation of short-run. The result is not strong and we can not confirm the purchasing power parity in the long run. This conclusion challenging many previous papers such as Aslan et al. (2010), who finding that Purchasing Power Parity held in both official exchange markets and black markets in Turkey.

For the second question, we can focus on the error correction in our case. We add the lag length to three but we can not find a significant data from the final result. The model makes little sense in the real economy, but if the equivalent relationship can not be found after the correction, there will exist no relationships in reality. In other words, since purchasing power parity does not even pass the error correction model, it will not hold in the short term. This is correspond to the deviation from purchasing power parity.

**5. Conclusion**

In recent year, the exchange rate market regime has stepped into a relatively stable regime. While a little countries implement the fixed exchange rate, most countries have a flexible exchange rate regime. However, the theory of purchasing power parity has a long history and many scholars test the theory by using past data far away from now. The question of consistency with the purchasing power parity in contemporary exchange rate markets received little supporting evidence. To fill the gap, this dissertation use the data of the exchange rate between the U.S. and Japan and the PPI in both country from 2009 to 2018 to test the purchasing power parity. This ten year is the recent ten year, and can explain the PPP theory under the present context. In addition, through assessing the short-run and long-run conditions of purchasing power parity, this dissertation hopes to make some contribution in the exchange rate policy determination and import and export markets.

The empirical evidence in our study does not support the purchasing power parity. It showed that there just exists statistic equilibrium between exchange rate and purchasing power in the long term. However, we can not explain the economic meaning and the afterward evidence confirms the proposition. No evidence support the short-run PPP between the U.S. and Japan.

The conclusion of this dissertation may contribute the development of export and import markets. First, the dissertation argues that it has no obvious equilibrium relationship between exchange rate and purchasing power. Since purchasing power can be related to the traded goods, exporter and importer can consider the result in this study to adjust to their own export or import policy. For example, if domestic purchasing power is relative higher, the importer can increase the magnitude of importing goods. On the other hand, the government should monitor cautiously the exchange rate markets with the theory of PPP. When the domestic economy is in a recession, the government can use the exchange rate to stimulate the economy. Otherwise, to trade off the international trade (import and export trade), government should adjust to the exchange rate appropriately.

The empirical confirms the deviation from PPP, therefore, it is recommended that policy makers do not have to worry about some fluctuations in the exchange rate markets.

There are several limitations in this dissertation. Firstly, this dissertation just focus on the flexible exchange rate regime. Johnson (1990) also test the PPP theory under fixed exchange rate. It is not suitable to deny the purchasing power parity just under the flexible exchange rate regime. Secondly, the data selected to represent the purchasing power may not be very appropriate. To test the PPP theory, purchasing power is an important variable we need to consider. However, the selection of index to represent the purchasing power has controversy until now. The different choice may have a effect on the result of our dissertation. At last, the size of our sample is not such sufficient. The relationship in PPP theory may be more significant if the sample is sufficient.

The dissertation analysed two developed countries, the structural shock in both country may be more obvious than other developing countries. On the other hand, the trade war has a higher possibility in developed countries, which may have some effect on the exchange rate markets. Therefore, it is recommended that future research assess the recent data among developing countries or between emerging country and developed country. Furthermore, based on the limitations, as purchasing power is hard to specify with index, future research should attempt to measure the purchasing power with multi index.